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Recognition of words from the EEG Laplacian

Reconhecimento de palavras com o Laplaciano do EEG

J. Acacio de Barros
Professor Adjunto do Departamento de Física – ICE,
Universidade Federal de Juiz de Fora
Juiz de Fora, MG 36033-330, Brazil

C.G. Carvalhaes
Professor Adjunto do Instituto de Matemática e
Estatística, Universidade Estadual do Rio de Janeiro
Rio de Janeiro, RJ 20550-900, Brazil
E-mail: carvalhaes@uerj.br

J.P.R.F. de Mendonça
Professor Adjunto do Departamento de Física – ICE,
Universidade Federal de Juiz de Fora
Juiz de Fora, MG 36033-330, Brazil

P. Suppes
Lucie Stern Professor of Philosophy,
Emeritus - CSLI – Stanford University
Stanford, CA 94305-4115, USA

Abstract
Recent works on the relationship between the electroencephalogram (EEG) data and psychological stimuli show that EEG recordings can be used to recognize an auditory stimulus presented to a subject. The recognition rate is, however, strongly affected by technical and physiological artifacts. In this work, subjects were presented seven auditory stimuli in the form of English words (*first, second, third, left, right, yes, and no*), and the time-locked surface electric potentials was recorded with a 64 channel Neuroscan EEG system and related to these stimuli. We used the surface Laplacian operator to eliminate artifacts due to sources located at regions far from the electrode and to improve the recognition rates of auditory stimuli from the surface electric potentials. To compute the Laplacian, we used a spline interpolation from spherical harmonics. The EEG Laplacian during stimulation were average over trials for the same auditory stimulus, and with those averages we constructed prototypes and test samples. In addition to the Laplacian, we applied Butterworth band-pass digital filters to the averaged prototypes and test samples, and compared the filtered test samples against the prototypes using a least squares metric in the time domain. We also analyzed the effects of the *spline* interpolation order and band-pass filter parameters in the recognition rates. Our results suggest a spatial isomorphism between both subjects in respect to the auditory word processing.
Keywords: EEG, *Spline* interpolation, Stimulus recognition, Surface electric potentials, Surface Laplacian.

Resumo
Trabalhos recentes sobre a relação entre o sinal de eletroencefalograma (EEG) e estímulos psicológicos mostram que o EEG pode ser usado para reconhecer estímulos auditivos apresentados a um sujeito. A taxa de reconhecimento, no entanto, é fortemente afetada por artefatos técnicos e fisiológicos. Neste trabalho, estímulos auditivos foram apresentados a dois sujeitos na forma de palavras inglesas (first, second, third, left, right, yes, e no), e o potencial elétrico de superfície foi gravado usando um equipamento Neuroscan de 64 canais e relacionado a esses estímulos. Aplicamos o operador Laplaciano de superfície para eliminar artefatos gerados por fontes localizadas em regiões distantes dos eletrodos de captação e melhorar as taxas de reconhecimento de estímulos auditivos dos potenciais elétricos de superfície. Para calcular o Laplaciano, empregamos um algoritmo de interpolação do tipo spline com harmônicos esféricos. O Laplaciano do sinal EEG durante a estimulação foi promediado sobre diversos segmentos do mesmo estímulo auditivo, e as médias foram usadas para construir protótipos e amostras de teste da palavra. Além do Laplaciano, aplicamos filtros digitais do tipo Butterworth aos protótipos e amostras de teste, e comparamos os testes filtrados com os protótipos, usando uma métrica de mínimos quadrados no domínio do tempo. Analisamos também os efeitos da ordem de interpolação e dos parâmetros do filtro digital nas taxas de reconhecimento. Nossos resultados sugerem a existência de um isomorfismo espacial entre os sujeitos estudados com relação ao processamento auditivo das palavras.
Palavras chave: *EEG, Interpolação spline, Potenciais elétricos de superfície, Reconhecimento de estímulos, Superfície Laplaciana.*

Introduction

Electroencephalogram (EEG) has been used widely to study processing of words by the brain. Evoked potential components, like the P300 or the N400, indicate collective macroscopic behavior of neurons related to speech processing. Recently, in a series of papers, Suppes and collaborators (Suppes *et al.*, 1997, 1998, 1999a,b; Suppes and Han, 2000) showed that surface electric potentials recorded via EEG techniques (and magnetic fields, via MEG – magnetoencephalogram) could be used to recognize auditory stimuli presented to a subject. These results indicate an isomorphism between EEG recorded brain signals and speech, therefore showing that some information about brain processing of speech is available at the field level. The recognition rates of the auditory stimuli from the surface electric potentials obtained by Suppes *et al.* (1997) were several standard deviations above chance recognition, but there is room for improvement, as their rates of correct recognition ranged from 37% to 97%. Improving such recognition rates is important, not only to understand the processing of information inside the brain, but also to be usable in practical applications. For example, the use of such surface electric potentials to control computers and equipment for disabled people would require reliable rates of recognition before they could be used in commercial products.

One of the possible causes for the misrecognition of words in the surface electric potentials is the presence of technical and physiological artifacts. For example, when a subject blinks, disturbances in the electric potential generated at the eye's muscles propagate to the scalp, affecting the EEG signal. In this paper, we use the Laplacian to emphasize the electric ativities that are spatially close to a recording electrode, filtering out those that might have an origin outside of the skull (Nunez, 1981; Nunez and Westdorp, 1994; Infantosi and Almeida, 1990). Our criterion for testing whether the Laplacian removes undesirable signals is the recognition rate of brain representations of spoken words.

Laplacian Computation

As mentioned earlier, some sources of noise in the EEG recordings of the scalp potential are bioelectric or other potentials generated outside of the scalp that propagate to the measuring electrode. For example, the generation of static electricity by the subject, the induction of currents from powerlines, and biopotentials that propagate from the eye movements, cardiac muscle or from other muscles are all sources that interfere with the measurements of the scalp potential produced by sources within the brain. In order to reduce these noises, it would be interesting to obtain, from the EEG, local information about the brain activity. One possibility is the surface Laplacian of the scalp potential, defined as:

$$\nabla^2_{surface} = \nabla_{surface} \cdot \nabla_{surface}, \tag{1}$$

where, in cartesian coordinates, using the unit basis vector perpendicular to the surface,

$$\nabla_{surface} = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y}. \tag{2}$$

The surface Laplacian of the scalp potential is a local operator with a simple physical meaning. Since the Laplacian of the electric potential is the divergence of the surface electric potentials, if we assume that the scalp is an ohmic conductor with conductance σ, then the current density **J** is linearly related to the gradient of the potential Φ by

$$\mathbf{J} = -\sigma\nabla\Phi, \tag{3}$$

where,

$$\nabla = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}. \tag{4}$$

Furthermore, if we assume that there are no sources of charge at the scalp but there are charges from the skull flowing to the scalp, the three-dimensional divergent of **J** is zero, but the surface Laplacian is not, as, from equation (3) and the definition of the surface Laplacian we have:

$$\nabla_{surface} \cdot \mathbf{J} = -\sigma\nabla^2_{surface}\Phi. \tag{5}$$

Thus, the surface Laplacian of the scalp potential is proportional to the local-flux of electric charge from the skull to the scalp. The surface Laplacian, therefore, is a locally measurable property of the brain electrical activity.

One of the practical problems to be solved in using the Laplacian for the recognition of speech via surface electric potentials is how to compute the second derivative of such a function, if only a finite number of experimental data points is available[1]. The simplest way is to use a discrete approximation in a two-dimensional surface grid. To compute the Laplacian at position

[1]In our case, only 57 out of 64 Neuroscan channels were used to collect data, as 7 channels were used as auxiliary channels. See below.

we need to compute the second-order derivative of the potential Φ on $\mathbf{r}_c$ with respect to x and y, the two coordinates defined to be orthogonal and parallel to the surface. We start with the discrete grid shown in Figure 1. Using the method of finite differences (Tierra-Criollo at al., 1997; Moin, 2001; De Moura, 2002), it is easy to show that the second derivatives of Φ with respect to x and y are, at the point, approximated $\frac{\partial^2\Phi}{\partial x^2} \approx \frac{\Phi(\mathbf{r}_1)+\Phi(\mathbf{r}_2)-2\Phi(\mathbf{r}_0)}{\Delta x^2}+O(\Delta x^2)$, and similarly for y, where Φ ($\mathbf{r}_i$)is the potential at point i, $i=1,2,3,4$, and $\Delta\boldsymbol{x}=(\Delta\boldsymbol{y})$ is the distance between detectors in the direction $x(y)$. Thus, the surface Laplacian of the scalp potential is computed at detector "0" (see Figure 1) as being simply

$$\nabla^2_{surface}\Phi \approx \frac{\Phi(\mathbf{r}_1)+\Phi(\mathbf{r}_2)+\Phi(\mathbf{r}_3)+\Phi(\mathbf{r}_4)-4\Phi(\mathbf{r}_0)}{\Delta l^2}, \quad (6)$$

where we used the approximation that $\Delta x = \Delta y \equiv \Delta l$. One of the problems with the discrete approximation is the error, due to the finite-size step, particularly, is high if Φ varies too much within the distance Δl. This is especially relevant if the source of interest is located between two electrodes.

To address the issue of having a discrete amount of information to compute an essentially continuous function (the Laplacian), it is useful to make continuity and smoothness assumptions on the functions and use interpolation techniques. It is natural to ask what kind of interpolation we can use to compute the surface Laplacian of Φ. A good candidate is the spherical spline interpolation, introduced by Perrin *et al.* (1987, 1989), as it yields results that are easily computed numerically.

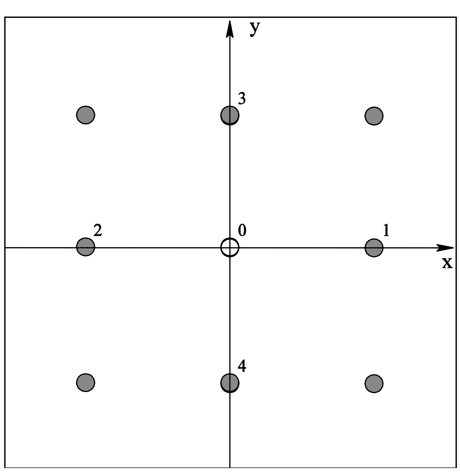


Figure 1. Grid showing nine EEG detectors and the coordinate system parallel to the surface, x and y. In this Figure we choose the origin of the coordinate system to coincide with the central electrode, labeled "0", whose coordinates are (0,0). Each small circle represents the position of an actual EEG electrode.

When doing an interpolation, we want to find out the best function, Φ ($\mathbf{r}$), that fits the finite set of data points, i.e., the points where we know the actual value of Φ . In other words, given n experimental values of $\Phi(\mathbf{r})$ at electrode positions $\mathbf{r}_i$ ($i=1,\cdots,n$), we want to find the continuous function $\Phi(\mathbf{r})$ that best fits those values. There are several different ways to interpolate $\Phi(\mathbf{r})$, but for simplicity in our computations we start with the assumption that the potential at point $\mathbf{r}$ can be represented as a supperposition of spherical harmonics as:

$$\Phi(\mathbf{r}) = c_0 + \sum_{i=1}^{N} c_i g_m\left(\frac{\mathbf{r}_i\cdot\mathbf{r}}{r_i r}\right), \quad (7)$$

where

$$g_m(x) = \frac{1}{4\pi}\sum_{n=1}^{\infty}\frac{2n+1}{n^m(n+1)^m}P_n(x), \quad (8)$$

$P_n(x)$ is a legendre polynomial on x of order n and $m>1$ is an integer called the *interpolation order*. We need to find the values of the coefficients c_i that best fit the experimental data. Let us define the following matrices.

$$C = \mathrm{diag}(c_1,\ldots,c_n),$$

$$Z = \mathrm{diag}(\Phi(\mathbf{r}_1),\ldots,\Phi(\mathbf{r}_n)),$$

$$G = \begin{pmatrix} g_{11} & g_{12} & \cdots & g_{1n} \\ g_{21} & g_{22} & & g_{2n} \\ \vdots & & \ddots & \vdots \\ g_{n1} & g_{n2} & \cdots & g_{nn} \end{pmatrix},$$

where $g_{ij} = g_m(\mathbf{r}_i\cdot\mathbf{r}/r_i r)$. It follows at once that the coefficients that better fit the data can be written in terms of the above matrices as

$$C = G^{-1}Z. \quad (9)$$

We are setting $c_0 = 0$ for two reasons. First, because c_0 is a global value added to a potential, and therefore in most cases irrelevant. Second, since we are interested in computing the derivative of the potential, this term will be thrown away regardless of its value. This reduces the problem to, computationally, finding the inverse to the matrix G and multiplying it by Z.

The reason for choosing Equation 7 for the potential is that, once we have the coefficients $c_{i'}$, it is straightforward to obtain the surface Laplacian. This is true because $\nabla^2_{surface}P_n = -n(n+1)P_n$.

Therefore, we obtain at once (Perrin *et al.*, 1989) that

$$\nabla^2_{surface}\Phi(\mathbf{r}) = \sum_{i=1}^{N} c_i h_m\left(\frac{\mathbf{r}_i \cdot \mathbf{r}}{r_i r}\right), \tag{10}$$

where

$$h_m(x) = \frac{1}{4\pi}\sum_{n=1} \frac{2n+1}{n^{m-1}(n+1)^{m-1}} P_n(x). \tag{11}$$

Equation 10, together with Equation 9, is the expression we use to implement the Laplacian of the potential in our computations.

Experiment and Data Processing

Data Acquisition

The EEG data set we used was collected by Suppes and collaborators. The data acquisition was described in details in Suppes *et al.* (1997), and here we will only reproduce the relevant information. For the data we used, corresponding to subjects S6 and S7 in Suppes *et al.* (1997), surface electric potentials were recorded using a 64-channel EEG system (NeuroScan, Herndon, USA) at the Palo Alto Veterans Affairs Health Care System. Our choice of subjects S6 and S7 was needed, as they were the only subjects in the experiment that had their surface electric potentials recorded, through the associated electric potential, with a 64-channel EEG system, since in all other subjects the international 10-20 electrode placement system was used. Subjects S6 and S7 were normal males, 75 and 30 years old, respectively. S6 was a native speaker of English, and S7 a native speaker of Chinese, but fluent in English. The electric potentials were recorded with reference to the linked earlobe electrodes. The data was filtered by a band pass filter ranging from DC to 200 Hz, and sampled at a rate of 500 Hz. Auditory stimuli, with durations of about 300 ms each, were presented to the subjects in intervals that varied from 1.5 to 1.7 s. The stimuli consisted of the seven English words: *first, second, third, yes, no, left, and right*. Words were presented in a random order, in a total of approximately 100 trials for each word. Subjects were instructed to listen to the words carefully.

Data Analysis

To compare the EEG surface potential to the Laplacian, we followed a procedure similar to the one described in Suppes *et al.* (1997), but we used the Laplacian computed at each electrode in lieu of the scalp potential. For each such potential a baseline was set by averaging the first 204 observations before the onset of stimulus, and then subtracting this average from each trial. Then we split the data into two sets, one with trials labeled even (E) and another with odd (O), and a prototype was created averaging all trials of a given word in one of the half sets, e.g. the E set. With the remaining trials, test samples were averaged with trials for each word. After prototypes and test samples were created, they were compared to each other via least square distances between the two averaged waveforms. A test sample was correctly classified from the surface potential if its least square distance to the correct prototype was the smallest one. In order to achieve the best recognition rate, we ran the above procedure for several different band-pass ranges of a Butterworth digital filter, in an attempt to filter out artifacts. To filter the data, we computed the FFT of each test sample and prototype using FFTW 3.0 (Frigo and Johnson, 1998). We then applied to the FFTs a fourth-order Butterworth filter and did an inverse FFT, returning to the time domain. The classification scheme used for the scalp potential is the same used by Suppes *et al.* (1997).

To classify using the Laplacian, we first computed the Laplacian for each electrode via spherical spline interpolation of order m, and with this new signal, we followed a similar procedure to that described in the previous paragraph. Finally, we made a search for the best classification filter and Laplacian interpolation m, searching what combination of values of the filter and m yield the best recognition rate.

Results and Discussions

Our main results are summarized in Table 1. We found improvements in the recognition rates by using the Laplacian. For example, for subject S6 the best recognition rate using the Laplacian was 88%, with a band-pass filter from 4.5 Hz to 6.5 Hz, whereas the best recognition rate using the scalp potential was 79%, with a band-pass filter from 2.0 Hz to 12.5 Hz. On the other hand, our worst Laplacian recognition rate was 62% for subject S7 in the composing scheme where the odd trials were used to create the prototypes and the even trials to create the testing samples, and it equalled that of the scalp potential recognition rate. Thus, for this small set of subjects we got consistently better results if we used the Laplacian for our classification scheme. For completeness, we show in Figure 2, the best rates of recognition from the surface electric potentials for different values of the band-pass filter. We can see that the Laplacian narrows the region of good recognition, if compared to the scalp potential.

Table 1. Highest recognition rates for each subject using the potential and Laplacian processing. Shadowed lines correspond to the Laplacian processing. The EO (OE) scheme refers to the analysis in which even (odd) trials are used to compose the prototypes and the odd (even) are used to compose the test samples

Subject	Composing Scheme	Highest recognition rate (%)	Parameters for the best results			
			Best EEG sensor	Best interpolation order (m)	Best filter (Hz)	
					low freq.	high freq.
S6	EO	79	F5	4	5.5	6.5
		76	C2A		5.5	6.5
	OE	88	C4A	3	4.5	6.5
		79	C3		2	12.5
	EO	66	C1P	4	1	4.5
		62	C6A		2.5	10
	OE	62	F7	4	1.5	22.5
		62	C6A		2.5	8

We also computed the recognition rate for different values of n, the number of average trials per test sample. Figure 3 shows the recognition rates as a function of n for the Laplacian and potential. We can see in Figure 3, from linear regression lines, that the Laplacian (solid line) consistently outperforms the potential (dashed line), but both yield approximately the same result with single trials. Even though Figure 3 shows only results for subject S6, both S6 and S7 showed the same behavior, with the Laplacian consistently outperforming the potential.

Since the Laplacian is a second-order spatial derivative (and, therefore, local), it is interesting to look at the spatial distribution of brainwave recognition rates on the scalp. Figure 4 shows the distribution for subjects S6 and S7, both for the Laplacian and the potential. We can see that for the scalp potential, S6's best electrode position is almost opposed to that of subject S7, with both best electrodes being global maxima. On the other hand, the Laplacian maps show three distinct local maxima for subjects S6 and S7 that have reasonably good recognition rates. Furthermore, we can see that for both subjects the localizations of those three regions seem to be similar. These data indicate that the best recognition loci for the Laplacian could be invariant among subjects, contrary to the scalp potential.

We also investigated the effects on the recognition rates when using the international 10-20 system of electrode placement instead of the 64-electrodes. This is an important question, as 10-20 systems are widely available and significantly less expensive (Figure 5 shows the spatial distribution of electrodes). For subject S7,

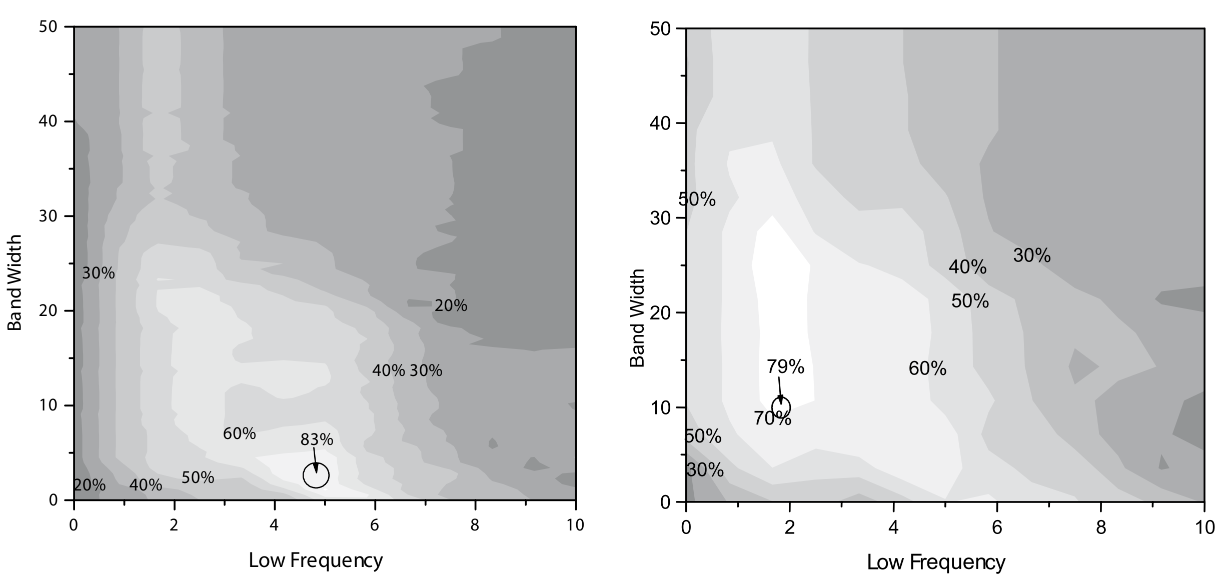


Figure 2. Distribution of recognition rates, for subject S6, with different filters. The left graph shows electrode C4A and the recognition was computed using the Laplacian. The right graph shows the recognition rates of using the potential at electrode C3.

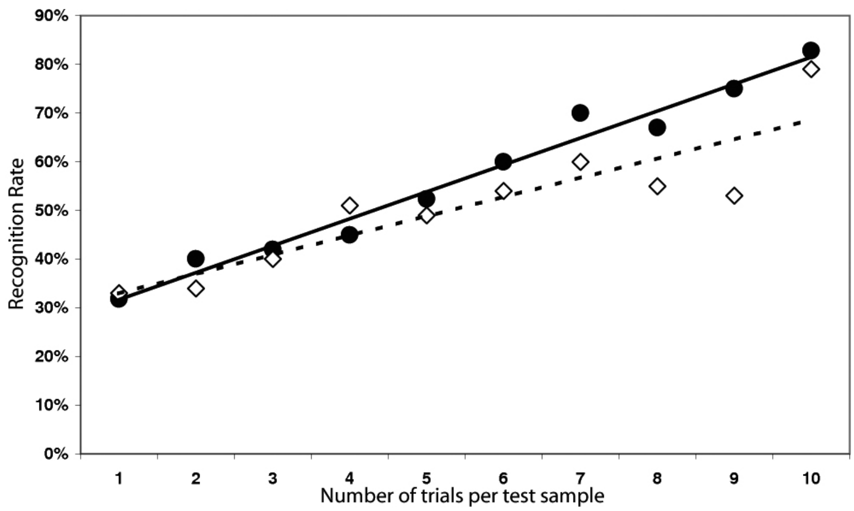
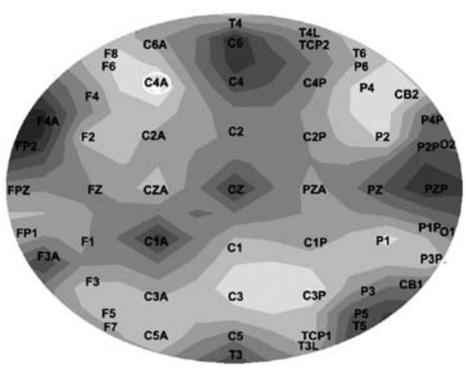
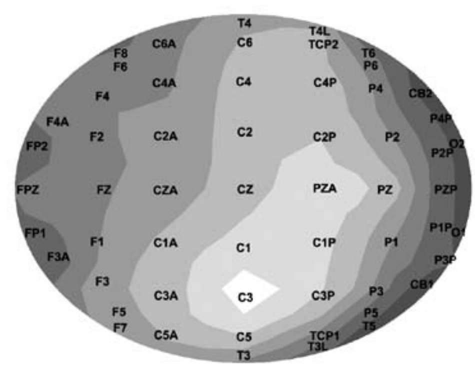


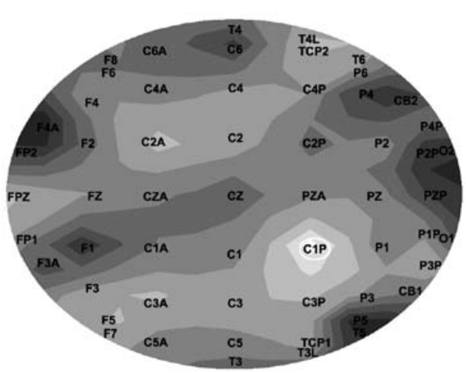
Figure 3. Recognition rate for different number of trials per test sample using the best filter for subject S6. The full circle represents recognition rates for the Laplacian, and the black line the best straight line fitting these data. The diamonds are the rates for the potential, with the dashed line representing their best linear fit. Both data are for OE configurations. The Laplacian corresponds to the position for electrode C4A and the potential to electrode C3.



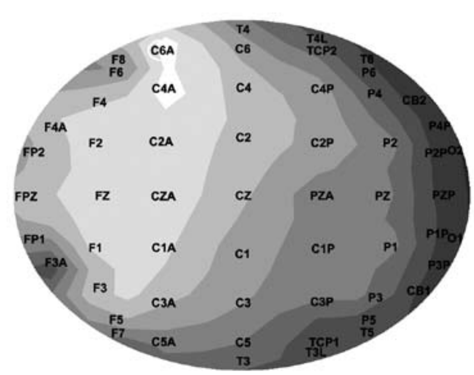
(a) Subject S6, third-order Laplacian interpolation, OE configuration and band-pass [4.5 Hz , 6.5 Hz].



(b) Subject S6, potential analysis, OE configuration, band-pass [2.0 Hz ,12.5 Hz].

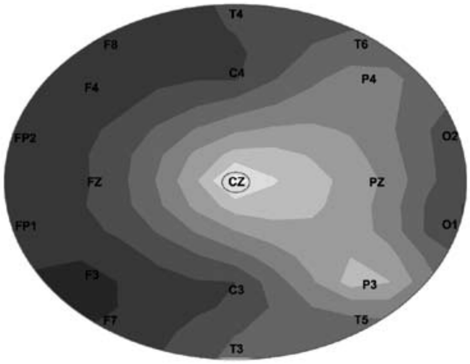


(c) Subject S7, fourth-order Laplacian interpolation, EO configuration and band-pass [1 Hz , 4.5 Hz].

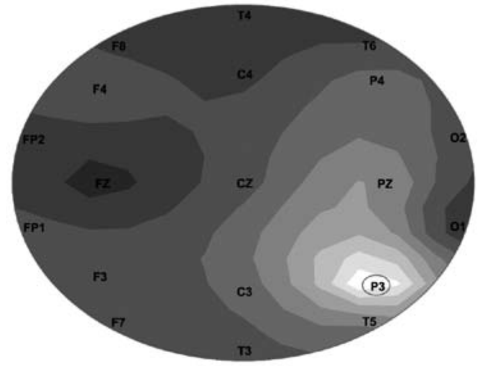


(d) Subject S7, potential analysis, EO configuration and band-pass [2.5 Hz , 10 Hz].

Figure 4. Distribution of recognition rates on the scalp for subject S6 and S7, considering the optimal filters shown in Table 1. The lighter areas show better recognition rates.



(a) Subject S6, fifth-order Laplacian interpolation, EO configuration and band-pass [4.5 Hz , 22.5 Hz].



(b) Subject S7, fifth-order Laplacian interpolation, EO configuration and band-pass [5 Hz , 9 Hz].

Figure 5. Distribution of recognition rates considering optimal filters for 10-20 system.

using only electrodes of the 10-20 system we got 79% as our best rate for the scalp potential, using electrode C3 with scheme OE. For the same subject and the same scheme, the Laplacian resulted in a recognition rate of 69%, inferior to the scalp potential. For subject S6, both the Laplacian and the scalp potential resulted in the same rates for the 10-20 system, namely 59%. The spatial distribution of best recognition rates for the Laplacian using a 10-20 system are shown in Figure 5. The decline in the performance of the Laplacian with the 10-20 system is a consequence of the wider spacing among the electrodes, which affects the interpolation procedure. Thus, the Laplacian improves the recognition rates only when we have electrodes that are not too far apart, as it is the case with the 64-electrode system.

Conclusions

We analyzed the experiment described by Suppes *et al.* (1997) using a similar least squares and filter search procedure with the Laplacian of the potential. The Laplacian was computed using a spherical harmonic spline interpolation method. Our case study indicates that, for the two subjects studied, the Laplacian gives better recognition rates than the original EEG signal for the 64-elecrodes schemes. It also indicates that this better result is maintained if we decrease the number of trials used to build the test sample.

In addition to better recognition rates, our Laplacian data suggest a possible important invariance result between subjects that is not present in the scalp potential. If we plot a distribution of recognition rates on the scalp, the Laplacian gives three loci of local maxima, and these three loci seem to be invariant between subjects S6 and S7. These points of local maxima also seem to be consistent with fMRI (Binder et al., 1997) and older neurophysiological results (Kandel and Schwartz, 1985) for speech processing areas (Broca's and Wernicke's areas).

Since we only have data available from two subjects, more data should be collected and analyzed to verify if the above results are robust. Unfortunately, the data collected by Suppes *et al.* (1997) used a 10-20 EEG system for most of the subjects, except for subjects S6 and S7. In order to use a smaller number of detectors in the Laplacian computation, we would need to confirm if the existence of the three loci of local minima remains when we downgrade from a 64 channel EEG to a 10-20 one. Figure 5 seems to indicate that they do not, i.e., 10-20 systems are not detailed enough to map the three local maxima. Since the Laplacian is a measurement of local surface current, the invariant loci of maxima suggest the existence of sources of information giving good recognition rates that should be independent. This result points to the possibility of using combinations of the Laplacian at these three points to improve the recognition rates obtained.

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