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LAPLACE's LAW OF THE HEART AND LEFT VENTRICULAR WALL THICKNESS

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<u>ABSTRACT</u>: Laplace's law applied to the heart predicts a thicker wall towards the base of the left ventricle than towards the apex. Visual inspection of anatomical dissections do not overtly show this difference, mostly because of important geometrical irregularities of the endocardium. The objective of this study was to find a pattern of distribution of wall thickness along the apex-base axis to demonstrate (or to reject) the prediction. With measurements made on eight human normal hearts, the data were fitted to a parabola of the type

$$h_r = B_o + B_1 l_r + B_2 l_2^2$$

where the coefficients B are equal, respectively, to 76.32, 0.77 and -6.64×10^{-3}, h_r is relative thickness and l_r is relative apex-base length. The correlation coefficient was 0.736 with a standard error of the estimate (SEE) of 8.85%. The maximum average thickness was located at 61% of the ventricular length (from the apex), with a region of "thicker wall" that went from 27.7% to 93.1% (roughly, from 30 to 90%) of that length. These results confirm the prediction of the law. Besides, it was found that this was true only, (1) for hearts in systole, and (2) when the papillary muscles were included as part of the wall thickness.

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INTRODUCTION

Pierre Simon de Laplace (1749–1827), French astronomer and mathematician, stated in his monumental work entitled **Traité de Mécanique Céleste** (published between 1799 and 1825) an expression relating surface tension, T [dynes/cm], on a membrane of negligible thickness separating two compartments, the difference of pressure, P [dynes/cm^2], between both compartments, and the two principal radii, R_1 and R_2 [cm], defining the portion or element of membrane considered, i.e.,

$$P/T = 1/R_1 + 1/R_2 = C_1 + C_2 \qquad (1)$$

where C_1 and C_2 are the principal curvatures.

The two principal radii describe on the membrane (or surface) two arcs of circumference perpendicular to each other precisely at their intersection, Q, where the tension T is tangentially defined. The surface of a volume of any shape can always be decomposed in many of these cap-like elementary pieces of membrane, including cases with opposite curvatures (the sign is then opposite).

Laplace did not offer a mathematical justification of his theorem or law, as it is now called. One of the simplest derivations (although leading to some conceptual difficulties) was given by Máximo Valentinuzzi (1939, 1950). A more rigorous demonstration can be found in the appendix of a paper by Sandler and Dodge (1963).

In most of the real cases of hollow organs or hollow elastic models, however, the membrane thickness has a well-defined not negligible value and, thus, we must introduce the wall-thickness, **h**. As a consequence, wall-stress, W_s [dynes/cm^2], can now be defined as surface tension, T, per unit wall-thickness, **h**, or

$$W_s = T/\mathbf{h} \qquad (2)$$

Solving eq.(2) for T and replacing it in eq. (1) leads to

$$P/W_s\mathbf{h} = 1/R_1 + 1/R_2 \qquad (3)$$

which becomes

$$P = 2W_s h/R \tag{4}$$

when $R_1 = R_2 = R$, that is, when the volume is a sphere of radius R. Usually, R (an equivalent radius) is taken as the mean value between the external and the internal radii. Eq. (4) is the commonest form of Laplace's law, as is frequently referred to in cardiac mechanics. In simple words, it states that the product of the hydraulic overpressure inside the heart and its equivalent radius is directly proportional to the (wall-stress)(wall-thickness) product.

The first application of this law in physiology (urinary bladder, uterus and heart) was made by Woods (1892). Máximo Valentinuzzi (1950), who started with the subject in the early 30's, performed extensive measurements in human uteri and in rubber balloons to test Laplace's law which, for historical reasons in the development of its particular application to this hollow organ, he called Barrau-De Snoo's law (because De Snoo had also studied the laboring uterus in the Netherlands, in 1936, while Barrau, a mathematician, verified and simplified the calculations). Burton (1957) revived it, making highly attractive considerations with respect to the heart. For example, he predicted that "midway up the ventricle, the wall must be thicker" (p. 804). However, anatomical dissections in different species do not overtly show this, mainly because of great geometrical irregularities of the endocardium. Consequently, the objective of the present paper was to demonstrate (or to reject) Burton's prediction. To the best of our knowledge, such study has not been made yet.

The rationale behind the prediction can be explained as follows: Let us assume that the left ventricle is a compound volume formed by two spheres, one of radius R_b (base) and a smaller one of radius R_a (apex). Since the intraventricular pressure (really, an overpressure with respect to outside the heart) is homogeneous all over its volume, it is seen that,

$$P = 2W_s h_b/R_b = 2W_s h_a/R_a \tag{5}$$

after applying eq. (4) to the basal and to the apical spheres, with wall-thicknesses h_b and h_a, respectively. Accepting that wall-stress is equal (assumption which may not be true), eq. (5) becomes

$$h_b/R_b = h_a/R_a \qquad (6)$$

saying clearly that h_b has to be greater than h_a for R_b is obviously always greater than R_a. In other words, the wall-thickness in the region of the base is expected to be thicker than in the apical region.

Preliminary gross measurements in several dog, cow, chicken and two human hearts indicated an upward convex distribution of wall-thicknesses when represented as a function of ventricular length, with the maximum about the middle part and closer to the base.

MATERIALS AND METHODS

Ten human hearts, obtained from cadavers, were used. They were found either in systole or in diastole. The criteria to differentiate systole from diastole were the macroscopical examination, the ventricular lumen (smaller in systole) and the wall thickness (bigger in systole). The hearts showed no lesion and the cause of death did not recognize any cardiovascular origin.

Each specimen was fixed in a 40% formol solution. The left anterior descending coronary artery (LAD) was dissected and clearly marked to be used as a constant reference location for all the cross-sectional segments. The length of the ventricles was measured from the atrioventricular sulcus to the apex, dividing this value in equal parts (usually 10 to 14, depending on the actual ventricular length). Thereafter, hearts were cut perpendicularly to the LAD with a slicing machine, obtaining 10 to 14 cross-sectional segments, as the case was (Figure 1).

A system of 12 polar axes was inscribed on a transparent sheet and this, in turn, was set, first, on the basal and, second, on the apical surface of each cross-sectional segment. Axis 12 was always laid exactly over the LAD cross-sectional cut and made it pass through the center of the lumen of the left ventricle (Figure 2). This center (which was approximate) was located by a simple geometrical construction: The angle subtended from the LAD point by the two tangents to the lumen periphery was drawn (t_1, t_2). Thereafter, the bisecting line was determined and the middle point between its two intersections (I_1, I_2) with that periphery was taken as the approximate center, O, of the left ventricular lumen at that level. Thus, the (6-12) axes were made to coincide with the bisecting line (Figure 2).

The thickness of the ventricular wall was measured as perpendicularly as possible to the wall at the site of each axis (1–12), on both surfaces (basal and apical), including the papillary muscles (second and third order, not first order muscles where the tendinous cords are attached) because embriologically these structures are part of the ventricular wall. Only those hearts in systole (eight out of the total of ten) were considered. The arithmetical average of these 24 measurements (12 apical of a given segment and 12 basal of the lower adjacent segment) was taken as the representative thickness of that level. At each level, the standard deviation (SD) of the 24 measurements was also calculated.

RESULTS

Plots showing the average ventricular wall–thickness, h_i, at each cross–sectional segment level, L_i, from apex to base were drawn for each heart. Figure 3a (papillary muscles included) illustrates the results for one particular heart. The upper curve is 1 SD above the lower curve (averages). It is easily seen that the maximum average thickness (slightly below 2 cm) appears between levels 5 and 6. Figure 3b shows the results for the same heart but excluding the papillary muscles from the measurements of thickness.

However, to summarize results, the data were pooled together and normalized. Of the total number n of segments, there were one or two at the apical region without lumen. These were discarded. Thus, the first segment (counting from the apex) was that one showing a full lumen at least on its basal surface. On the basal side, due to the anatomical distribution and insertions of the cardiac valves and other structures, there were also one or two segments with incomplete lumen. These were discarded too. Therefore, we defined as valid ventricular length, L_v, the distance between the apical and the basal levels where wall-thickness measurements could be made over all the 12 polar axes. Each cross–sectional segment had a length (segment length, l_s) and the position of each level was calculated as a percentage of L_v, considering as zero the first apical level with a full set of measurements. Wall–thickness was expressed as percentages with respect to the maximum value. Obviously, the normalization procedure reduced the total number of effective cuts.

Figure 4 displays the normalized results, i.e., relative ventricular wall-thickness, h_r, as a function of relative ventricular length, l_r, from apex A to base B. Each point is the average for that particular level. A total of 63 averages, obtained from 8 hearts, are represented. A parabolic curve fitting was calculated with the least-squares criterion (curve **a**) obtaining the equation,

$$h_r = B_o + B_1 l_r + B_2 l_r^2 \qquad (6)$$

with the coefficients B equal to 76.35, 0.77 and -6.64×10^{-3}, respectively, and the thickness and length expressed in percent, as explained above. The coefficient of correlation, r, was 0.736, and the upper and lower curves (**b**, **c**) indicate the standard error of the estimate (SEE = 8.85%), that is, 68% of the points must be contained within the band determined by **b** and **c**. In this plot we have 45 points within the band, representing 71% of the total population of 63 values.

DISCUSSION

Visual inspection of the longitudinal distribution of ventricular wall-thicknesses shown in Figure 4 puts into evidence a region of maxima. Eq. (6) leads to l_{rmax} = 61% as the location of the maximum average thickness, slightly shifted towards the base, as expected. We can define a region of maximum wall-thickness by projecting the maximum, M, to the left and to the right until this horizontal line intercepts curve **b** (points P and Q). In this case, the "thicker region" spreads from 27.7% to 93.1% of the apex-base axis. Another criterion could be to take the maximum value M minus 5% and drawing a horizontal line. Its intersections R and S with the curve **a** define also a region of "thicker wall", now going from 31.1% to 84.2% of the apical-basal axis. The difference of results between both criteria is minor.

Measurements were also made without considering the papillary muscles. Interestingly enough, the distribution of wall thicknesses along the longitudinal axis did not follow neither the expected pattern nor any other consistent one. This may be taken, then, as complementary evidence for the embriological concept that the papillary muscles are part, indeed, of the ventricular wall and must be included in the evaluation by Laplace's law.

In diastole (only two hearts), there was no apparent consistent distribution of wall-thickness versus ventricular length. We might speculate that this law should be valid only during the active period (systole). Nontheless, it does not look as plausible because the law was derived for passive systems. On the other hand, hemodynamically speaking, systolic wall-stress is regulated by the alteration in wall-thickness (Li, 1986). Obviously, more data are needed.

The septum may pose a problem. For these measurements we took the full thickness, i.e., from left ventricular endocardium to right ventricular endocardium. It is not likely that only one portion of the septum thickness acts for one ventricle and the remaining thickness for the other. The results seem to support this view. In this respect, we should remember that, in its normal operation, the left ventricle rather "drags" the right side, probably without an actual effective action of the latter.

If we consider eq. (4), this can be more generally rewritten as the product PR being proportional to the product $W_s h$, or

$$PR = kW_s h \qquad (7)$$

from which,

$$R/h = k\ W_s/P \qquad (8)$$

The ratios R/h and W_s/P are coincident with the pi-numbers pi_1 and pi_2 theoretically obtained by Li (1986). According to this author, these dimensionless numbers are invariant implying that Laplace's law applies to all mammalian hearts. Besides, experimental measurements of R/h by echocardiography are quite feasible, providing a means to estimate the left-hand side of eq. (8). Since pressure P is easily determined, this would permit in turn an estimate of the product kW_s as a measure of wall-stress. In principle, this should be applicable at any instant of the cardiac cycle. In diseased ventricles, changes in kW_s might be expected.

The orthogonality of the thickness measurements cannot be guaranteed due to practical reasons originated in the irregular geometry of the endocardium. It is difficult to establish a definite criterion. This acts as an interference which is partially overcome by the averaging over the cross-sectional periphery. Longitudinal profiles could be another intersting piece of information. However, endocardial irregularities are in this case extremely difficult to deal with.

In conclusion: Burton's prediction, based on Laplace's law, was found to be correct, that is, the left ventricular wall is thicker around the middle region and toward the base. This confirms also Li's assertion (1986) that Laplace's law links the anatomical design to the functional capability of the mammalian heart. However, it was found that the law was true only, (1) for hearts in systole, and (2) when the papillary muscles were included as part of the wall thickness.

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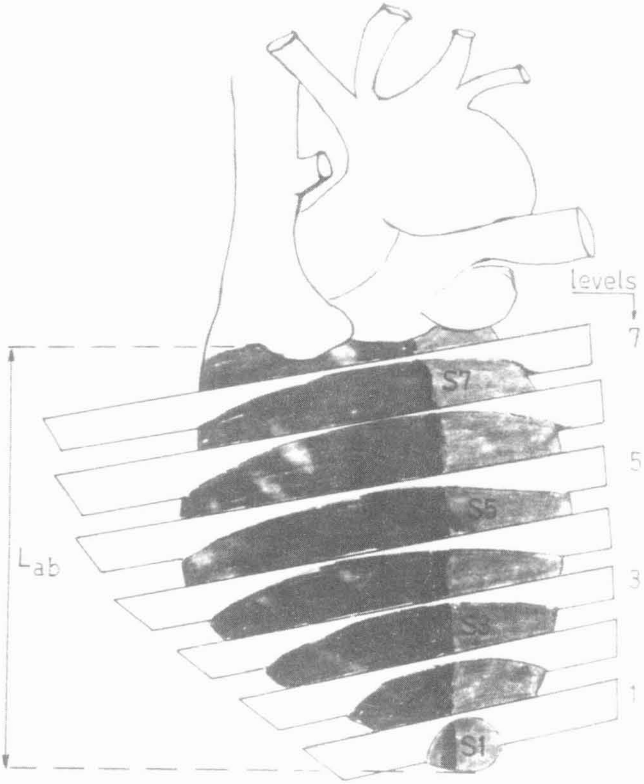

FIGURE 1

Cross-sectional slicing of the ventricles

Cuts were made perpendicularly to the course of the left anterior descending coronary artery (LAD), following the apical-basal length. Cross-sectional slicings defined segments. Each segment has a length delimited by the upper and lower levels. Besides, the representative wall thickness at any level was obtained by measurements made on the apical surface of the segment and on the basal surface of the lower segment.

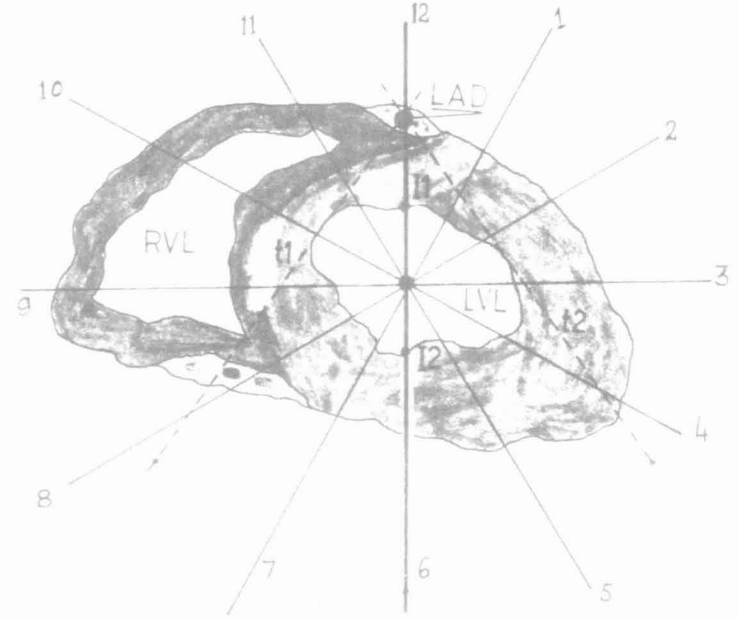

FIGURE 2

Cross-sectional segment of the ventricles seen from the apical surface

LAD: cross-section of the left anterior coronary artery. Axes (6-12) were determined by the bisecting line of the LAD-tangents angle (tangents subtended from LAD to the periphery of the left ventricular lumen, LVL). The center,O, of this polar system was taken as the midpoint between the intersections of the bisecting line with the LVL periphery. Thickness measurements were made with a vernier as perpendicular as possible to the wall at the location of each axis. RVL: right ventricular lumen.

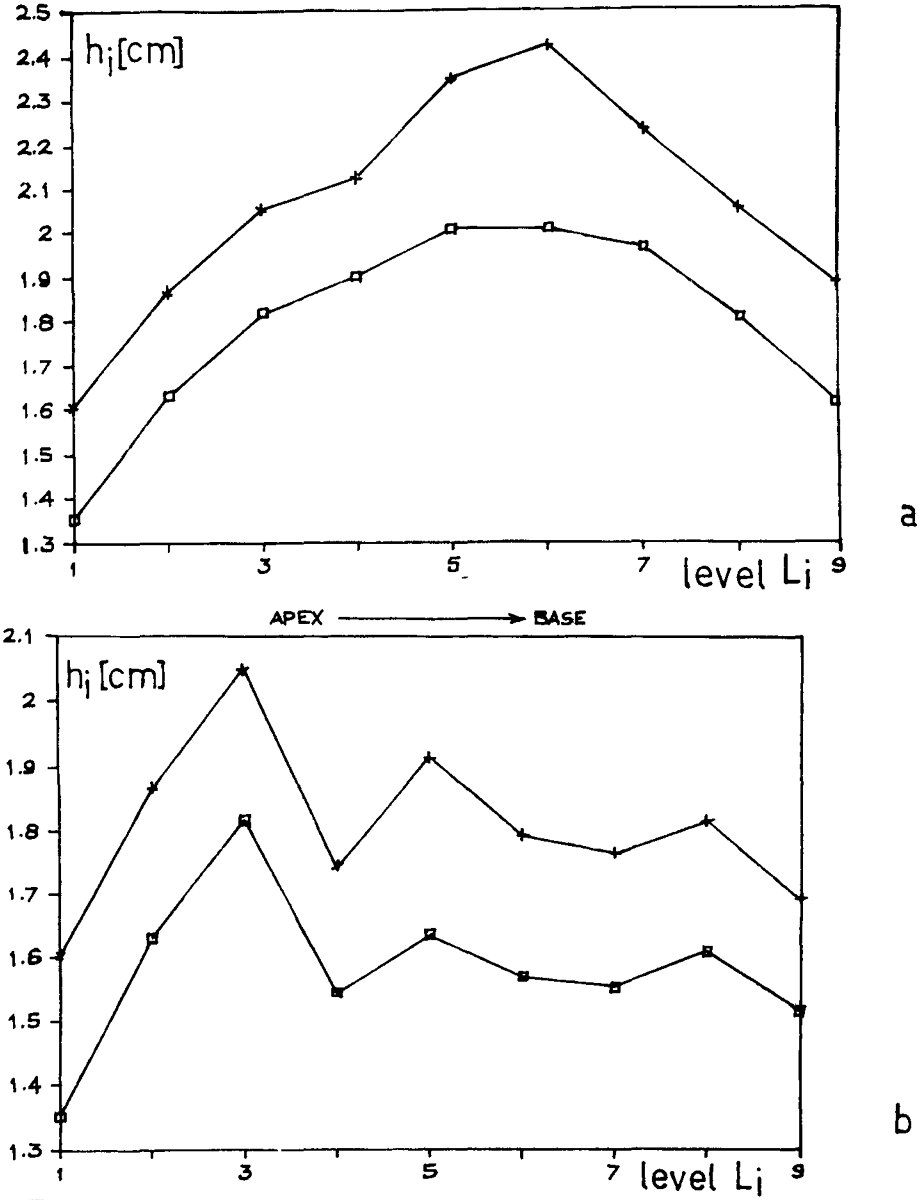

FIGURE 3
Absolute ventricular wall thickness as a function of the cross-sectional level from apex to base

Lower curves (squares): average values over 24 measurements per level. Upper curves (crosses): average plus one standard deviation. (a) Considering the papillary muscles as part of the wall. It conforms to the expected pattern. In this specimen, the maximum average was 2 cm between levels 5 and 6. (b) Excluding the papillary muscles. It does not conform to the pattern.

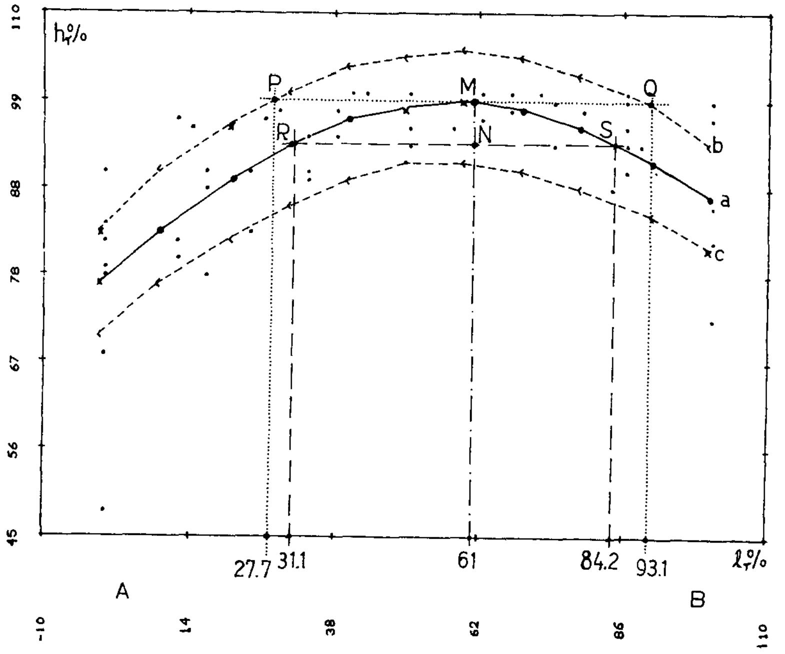

FIGURE 4

Relative ventricular wall-thickness as a function of the relative apical-basal axis

The origin (0%) corresponds to the apex, A, and the base, B, is taken as 100%. Points PQ define on the horizontal axis the region of thicker wall. By subtracting 5% from the maximum M, points RS define a slightly shorter region of thicker wall. After curve fitting, the maximum thickness M was found, for 8 normal hearts and 63 average points, to lie at 61% of the length measured from the apex. Curves **b** and **c** define the plus and minus 1 SEE band (8.86% above and below **a**).

LEI DE LAPLACE PARA ESPESSURA DE PAREDE DO CORAÇÃO OBTIDO E VENTRÍCULO ESQUERDO

RESUMO--A lei de Laplace aplicada ao coração prevê uma parede mais espessa em direção à base do ventrículo esquerdo do que ao ápice. Inspeção visual de dissecações anatonômicas não demonstra esta diferença, principalmente por causa de irregularidades geométricas do endocárdio. O objetivo deste estudo foi de encontrar um padrão de distribuição da espessura da parede ao longo do eixo base-ápice para demonstrar (ou rejeitar) a previsão. Com medições realizadas em corações de oito pessoas normais os resultados foram ajustados por uma parábola do tipo

$$h_r = B_o + B_1 l_r + B_2 l_r^2$$

onde os coeficientes B são iguais, respectivamente, a 76.32, 0.77 e $-6.64x10^{-3}$, h_r é espessura relativa e l_r é o comprimento relativo base-ápice. O máximo das médias de espessura foi localizado em 61 % do comprimento do ventrículo (a partir do ápice) com uma região de "parede mais espessa" que variou de 27.7% a 93.1% (aproximadamente de 30% a 90%) daquele comprimento. Estes resultados, confirmam a previsão da lei. Além disto, foi encontrado que isto é verdade somente, (1) para corações em sístole, e (2) quando os músculos papilares fossem incluídos como parte da espessura da parede.