RBE. VOL.6 N.2 1989

THE WIGNER DISTRIBUTION AND ITS APPLICATION TO NON-LINEAR ACOUSTIC WAVE PROPAGATION

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ABSTRACT -- The Wigner Distribution (WD) is a signal transformation allowing the simultaneous study, in both the time and frequency domains, of some characteristics of the process of which this signal is representative. It is particularly useful for the study of non-stationary signals. Non-linear acoustic wave propagation has become a source of great concern in medical ultrasound because of the suggested possibility of damage to tissues insonated with commercial medical ultrasound equipment. Its main relevant characteristic is the change in the frequency contents of the acoustic wave as it propagates along a medium with the generation of steepening wavefronts. This paper shows the applications of the WD to the study of finite-amplitude effects of ultrasound propagation in water.

INTRODUCTION

The use of ultrasound equipment in medical diagnosis and therapy has increased continuously in the past few years. Although many other imaging devices and techniques have come to light, the role of ultrasound methods in the assessment of many diseases and in therapy is bound to grow. Many researchers have observed finite-amplitude effects in tissues and other biological media (Dunn *et al.*, 1982; Muir and Carstensen, 1980; Starritt *et al.*, 1985). In their paper, Starritt *et al.* demonstrated non-linear propagation effects when using both ordinary pulse-echo diagnostic and continuous wave therapeutic equipment.

The study of non-linear ultrasound propagation is normally carried out using a point-like hydrophone, which introduces diffraction effects that must be unravelled from the non-linear effects (and sometimes from the further complications arising when focused transducers are used for generating the ultrasound field). We have previously presented (Costa *et al.*, 1987-a,b) a new technique based on a purpose-built PVDF hydrophone which enables the study of ultrasound fields without being compromised by the diffraction and focusing effects inherent in most measurement schemes. In those papers the non-linearities of the ultrasound field produced by focused transducers could be better understood and visualized when using our measurement technique, rather than the conventional approach.

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In this paper, a signal transformation is used in order to study finite-amplitude effects of ultrasound waves in ordinary tap water. This signal transformation is the so called Wigner Distribution (WD), and is relatively unknown in medical ultrasound applications. The WD allows the simultaneous display and analysis of both the time and frequency representations of a signal. It is particularly useful for analysing non-stationary signals, such as non-linear ultrasound pulses. It is shown how the non-linearities develop with range, and most interestingly, that they may be confined to a particular region in the time domain. Although this could be inferred from our previous paper (Costa *et al.*, 1987-a), the explicit visualisation of this result is made possible only by the WD.

THE WIGNER DISTRIBUTION

The WD was first introduced by Wigner (1932) in the context of quantum mechanics, and for signal analysis by Ville (1948). It makes it possible to show, in a single display, both the temporal evolution and the frequency content of a signal. The WD of a signal is given by:

$$W_f(t,\omega)=\int_{-\infty}^{\infty} f(t+\tau/2)f^*(t-\tau/2)\exp(-j\omega\tau)d\tau \qquad (1)$$

where:

t is the time variable

ω is the frequency variable

f^* denotes the complex conjugate of the signal $f(t)$

The Wigner Distribution has several very interesting properties and the reader should refer to the papers by Claasen and Mecklenbrauker (1980-a,b,c), Marinovic and Smith (1984) and Boudreaux-Bartel (1985) for further details, as well as to the original papers by Wigner and Ville. Some of the properties that are important for the work presented here are summarised below:

1- The WD of any real or complex signal will be real;

2- The WD of a real signal is an even function of the frequency;

3- The integral of the WD over the frequency variable at a certain time t yields the "instantaneous signal power" at that time, i.e.:

$$|f(t)|^2=\frac{1}{2\pi}\int_{-\infty}^{\infty} W_f(t,\omega)d\omega \qquad (2)$$

4- The integral of the WD over the time variable at a certain frequency ω yields the power spectral density at that frequency, i.e.:

$$|F(\omega)|^2=\int_{-\infty}^{\infty} W_f(t,\omega)dt \qquad (3)$$

From the above mentioned properties, and from the definition of the WD, it can be appreciated why it is said that this representation contains both the temporal and spectral features of the signal $f(t)$. This representation is now used in order to analyse non-linear ultrasound pulses.

THE EXPERIMENTAL SETUP

The experimental setup is shown in Figure 1. A pulsed ultrasound field was produced by a 19 mm diameter, 2.25 MHz (nominal) centre frequency, 9 cm (nominal) focused commercial PZT transducer (Philips). The transducer was attached to the wall of a tank filled with ordinary tap water at 19°C. The transducer excitation was achieved by applying a short square-wave pulse as the modulation waveform of a 2.3 MHz carrier provided by a signal generator, whose output was amplified by an ENI wideband power amplifier (50 dB over the frequency range .1 to 10 MHz). The output of the amplifier was then applied to the transducer. Controlling the input voltage to the power amplifier made it possible to generate ultrasound pulses both in the linear and in the non-linear ranges. For the linear case, the input voltage was set to 150 mV (peak-to-peak), while the non-linear ultrasound field was produced by a voltage of 830 mV (peak-to-peak). The trigger for the digital oscilloscope was provided by a time-delay pulse generator, triggered by the modulating signal applied to the signal generator. This allowed for the correct time-delayed ultrasound pulse be displayed on the oscilloscope screen, to be digitized and transferred to the computer (Compaq 386-20). The ultrasound pulse was measured with a purpose-built PVDF planar hydrophone, with a 75 mm diameter active surface (in order to ensure that the ultrasound field is entirely intercepted by the membrane), thereby eliminating unwanted diffraction and focusing effects in the field measurements (Costa *et al.*, 1987-a). The hydrophone was oriented perpendicularly to the pulse travel direction. The measurements were taken at several distances from the tranducer face, starting from 2 cm, and spanning to 27 cm.

EXPERIMENTAL OBSERVATIONS

Figure 2 shows the following time domain pulses: (a) linear case; (b) non-linear case, at the same measurement locations. The linear and the non-linear pulses show neither diffraction nor focusing effects, although it is quite easy to note that the latter vary with distance, as would be expected under such conditions. A better insight into the phenomenon can be gained with the help of the Wigner distribution. The WD was computed for all signals shown in Figure 2. Figure 3 shows contour plots describing some of the linear waveforms, and in Figure 4 the corresponding non-linear waveforms. The WD of the linear pulses show no (or very little) variation, simply confirming their invariance when measured with our diffraction-insensitive hydrophone in a linear field. In fact, the WD shows that our supposedly linear pulse is very mildly non-linear: the usual checks for non-linearity failed to uncover such a small effect, and this underlines the sensitivity of the technique. In a field developing non-linearities, however, the advantage of the use of our hydrophone is patent (Costa *et al.*, 1987-b), and the WD now provides a completely new way of looking at this phenomenon. Near the transducer, 2 cm from its face, the WD shows a very well behaved pulse, centred around 1.9 MHz, quasi-gaussian shape. As the measurements are taken further away from the transducer face, more and more higher frequency components start developing. Even more interesting is the possibility of noticing when, in the time domain, the non-linearities develop. As it can be seen, there is a gradual build up of harmonics (in power) around 2μs, and looking at the time domain pulses in Figure 2, this can be associated with the occurrence, in time, of a sharpening of the main positive peak, i.e the transition from a state of rarefaction to sudden compression, giving rise to the familiar shock-like wavefront in a non-linear field. This is further corroborated by a similar gradual building up of harmonics in the second main transition from a large negative peak (rarefaction) to a positive peak (compression). Another important point to be noted is that the pulse gradually decreases in amplitude, now due to the attenuation in water, eventually leading to a reduction of observable non-linear effects.

The analysis carried out in the previous paragraph was made possible only with the aid of the Wigner distribution. The main point was the concentration of the development of harmonics to specific "time regions" in the pulses. Both the starting and the ending of the pulses appear unspoiled by non-linear effects. This is probably due to the amount of power necessary to trigger the non-linear events and, as the ultrasound wave passes through the hydrophone and is detected, the initial variations in pressure are not sufficient to develop non-linearities in the field. Only when there is a huge difference in pressure from a region of rarefaction to one of compression (and this occurs in a short time), are we able to observe the building up of harmonics. The WD allows us to define exactly where the non-linear effects are occurring *within* the pulse. In general, it is not the whole waveform that is distorted, but only certain sections of it that exhibit finite amplitude effects.

CONCLUSION

We presented a relatively unknown signal processing tool applied to the study of finite amplitude effects in ultrasound wave propagation. This tool, the so called Wigner distribution, allowed us to gain a new insight into the development of harmonics in a pulsed ultrasound wave, showing that they are confined to certain "time zones" within the wave. The use of the WD in conjunction with the diffraction-insensitive hydrophone developed in our laboratory, provides a particularly useful experimental procedure for analysing these effects. With the growing concern about the safety of diagnosis and therapy with ultrasound, the possibility of assessing the output of ultrasound equipment is of foremost importance. With the use of the correct measurement technique and signal analysis, much can be learned in this regard, as suggested in this paper.

ACKNOWLEDGEMENTS

E.T. Costa wishes to thank CAPES (Coordenacão de Aperfeicoamento de Pessoal de Nivel Superior) of the Brazilian Ministry of Education, for financial support.

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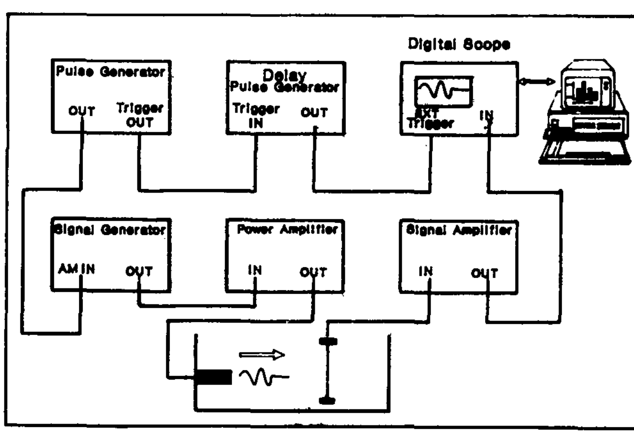

Figure 1: Experimental setup.

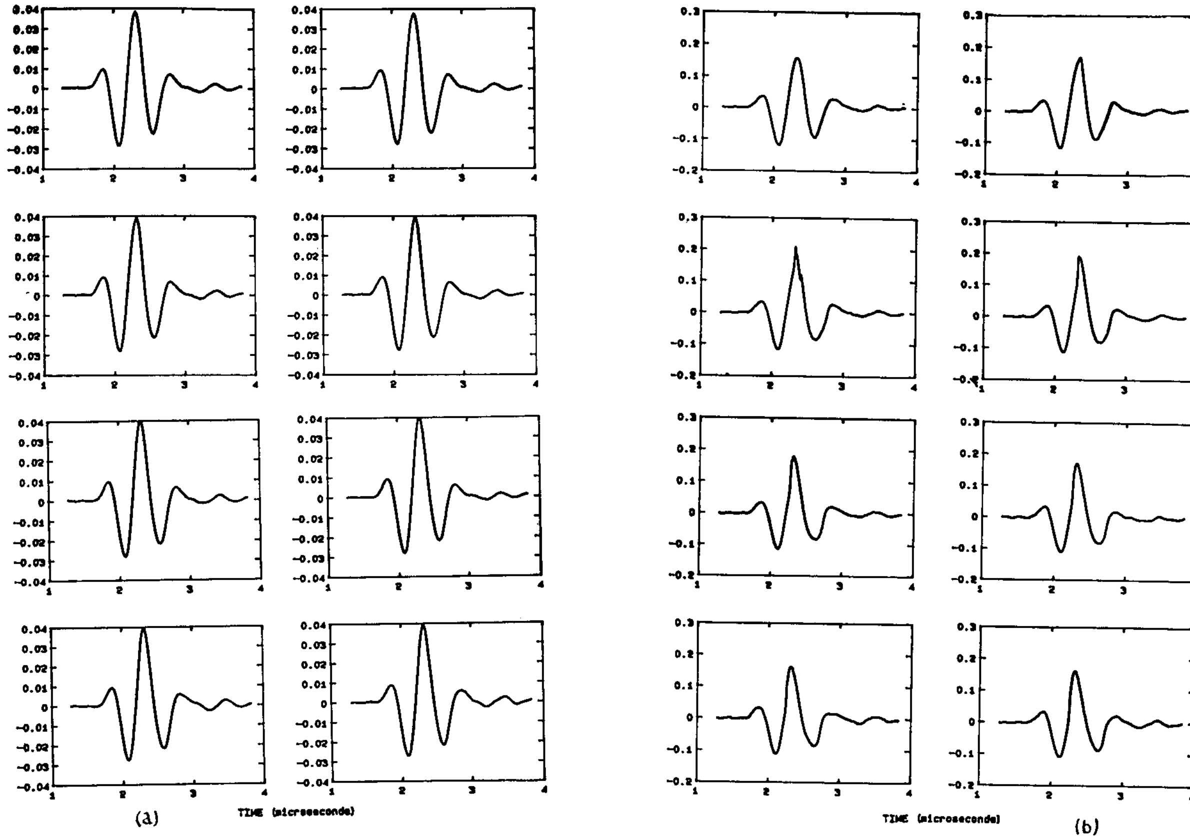

Figure 2: Time domain pulses, (a) linear and (b) non-linear.

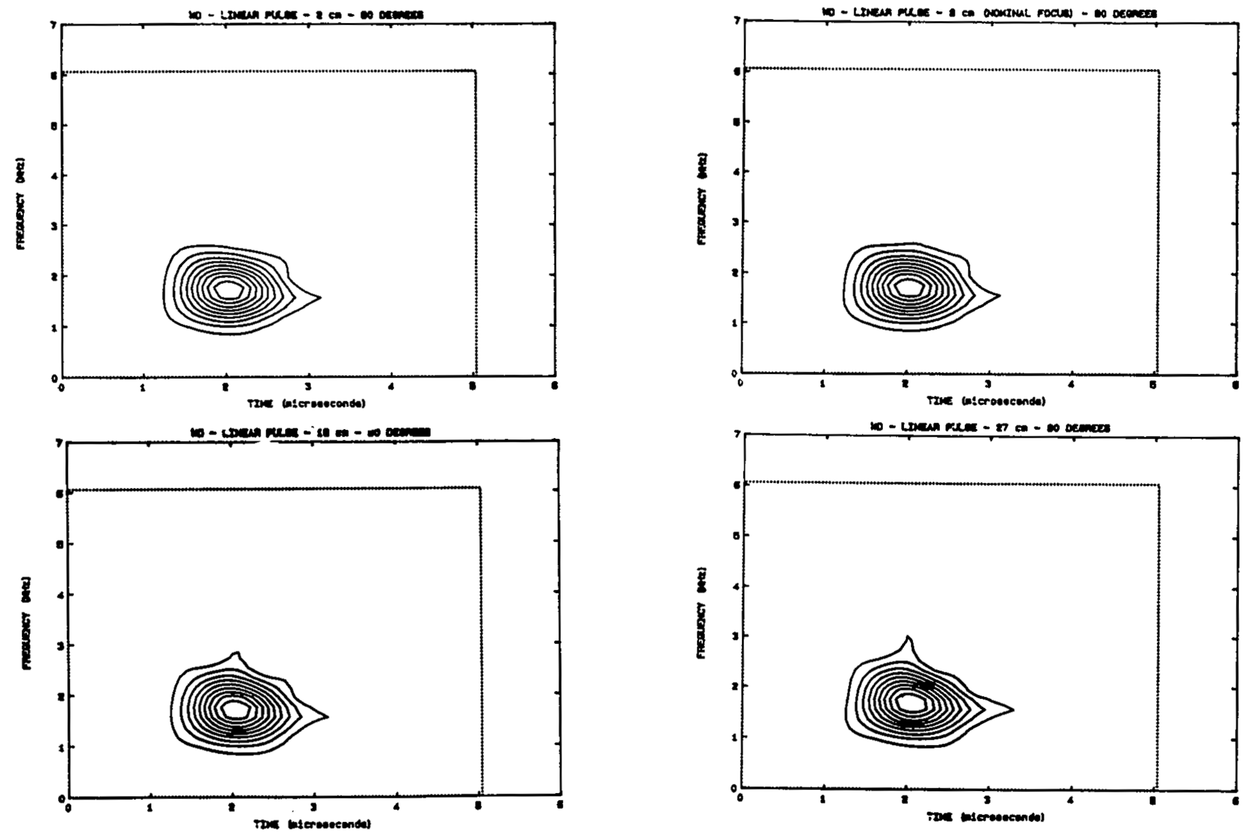

Figure 3: Wigner distribution of some of the waveforms shown in Fig. 2a (linear).

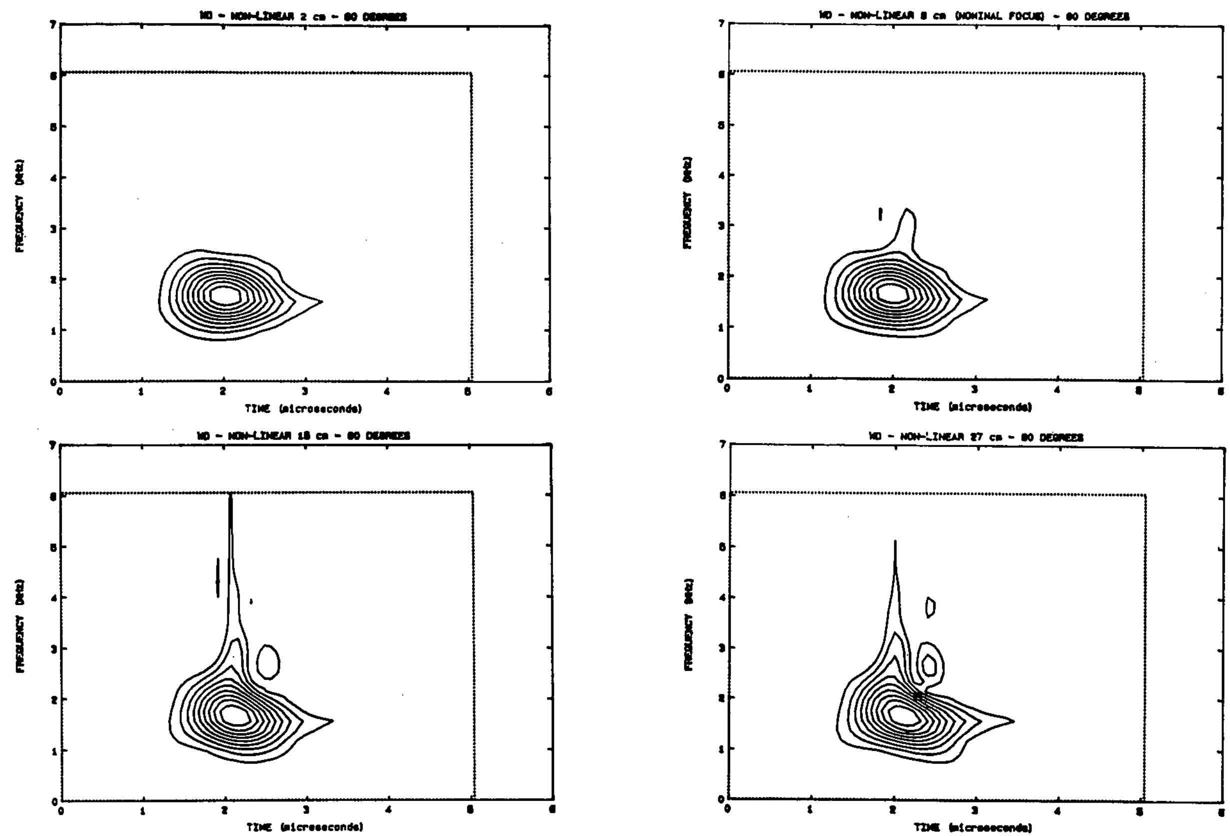

Figure 4: Wigner distribution of some of the waveforms shown in Fig. 2b (non-linear).