THE APPLICATION OF THE WIGNER DISTRIBUTION TO THE ANALYSIS OF EEG SIGNALS

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ABSTRACT – The need for a two-dimentional approach to the analysis of EEG signals is identified. The Wigner Distribution is proposed as a suitable candidate and a brief review of its major features is presented. The application of this algorithm to EEG records is described both in terms of the practical implications and the results obtained.

INTRODUCTION

The application of computer aided techniques to the analysis of EEG records poses many problems. The signal itself is low-level (150 μ V typ.), non-stationary and subject to significant corruption (usually in the form of muscle artefact). Further, consider able variations exist between different patients and within a record from a single individual.

Computer Assisted Diagnostic Systems (CADS) have been developed to help the electroencephalographer particularly in the analysis of long-term EEG recordings of a large population of epileptic patients. These involve elements of quantitative analysis and pattern recognition, or both. The three main goals of such CADS are the detection of interictal epileptiform events, the detection of epileptic seizures, especially petit mal absences, and the localization of epileptogenic areas of the brain (Niedermeyer & Lopes da Silva, 1987).

A good example of pattern recognition is in the detection of the epileptiform event known as a spike-and-wave complex (SAWC), which characterizes attacks of petit mal epilepsy. Many methods have been proposed for such kinds of detection (E.g. Gotman & Gloor, 1976; Comley & Brignell 1981; Stelle & Comley 1989), which have met with varying degrees of success, but all suffer from the recurrent problem relating to the difficulty in defining a suitable model with which to describe the features of interest. These problems are compounded by the fact that trained electroencephalographers find it difficult to describe in any formal way, the processes and criteria they employ in their manual analysis.

One fact which has become increasingly apparent to us is that human interpretation is based on some form of simultaneous time and frequency analysis. The above examples of

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computer aided methods are based on both the waveform and the spectrum of the signal, but considered separately (i.e. in one dimension).

The above would suggest that a much more reliable EEG analysis may be obtained from a consideration of both the time-domain and the frequency-domain features of the signal simultaneously, in a two-dimensional way. The most basic tool that permits such kinds of analysis is the spectrogram. This is most appropriate for analising long segments of EEG (quasistationary signals) but is not particularly useful for the analysis of a single epilepti form event such as a SAWC, mainly because of the low number of samples and the necessity of using windows that corrupt the original signal. Another special tool that shows the variations of the signal in both the time and frequency domains is the Wigner Distribution (WD). This has already been applied successfully in such areas as the analysis of ultrasound signals (Costa & Leeman, 1989). It is particularly efficient in the analysis of non-linear signals, as is the case of epileptiform events. Some examples of the application of the Wigner Distribution to the analysis of EEG signals will be shown in this paper.

THE WIGNER DISTRIBUTION

The Wigner Distribution, defined by (1), was introduced by Wigner in the area of Quantum Mechanics in 1932 and applied in signal analysis by Ville in 1948. It remained in obscurity until 1980, when Claasen and Mecklenbrauker gave it a new lease of life.

$$W_x(t,w) = \int_{-\infty}^{+\infty} x(t + \tau/2).x^*(t - \tau/2).\exp(-jw\tau).d\tau \quad (1)$$

where:

a) "t" represents time,
b) "w" represents frequency and
c) "x*(t)" represents the complex conjugate of x(t).

The WD offers several interesting properties, four of the more important are listed below. Properties 3) and 4) are of particular importance in this work. Property 3) permits flexibility as to where the feature to be identified is positioned within the time window. The significance of property 4) is that it represents a corruption of the output, the consequences of which will be seen later.

1)The WD of any signal (real or complex) is real.
2)The WD of a real signal is an even function of the frequency.
3)If the signal is shifted in time, the WD is also shifted in time.
4)The WD of the sum of two signals "f" and "g" is a bilinear function of "f" and "g" , or

$$WD(f+g) = WD(f) + WD(g) + 2.Re[WD(f,g)] \quad (2)$$

As with the Fourier Transform, the definition given by (1) is not useful in practical terms. In order to apply the WD to discrete-time signal processing and to calculate it via FFT techniques, equation (2), that represents the Discrete Time Wigner Distribution and (3), the Pseudo Wigner Distribution, may be used. It should be noted that the last one causes some blurring in the frequency domain.

$$W_x(n,w) = \sum_{m=-\infty}^{\infty} x(n+m).x^*(t-m).\exp(-j2wm) \qquad (3)$$

$$W_x(n,w) = \sum_{m=-\infty}^{\infty} x(n+m).x^*(t-m).h(m).h^*(-m).\exp(-j2wm) \qquad (4)$$

As expected, aliasing is caused by the sampled signal, but, as can be seen from (3) and (4), the repetition period is " π " rather than the "2π " that all spectra of the discrete-time signals have. For this reason, the signal should be sampled at least, at twice the Nyquist rate ($fs \geq 4.fmax$). This, however, is not a good idea in practical terms, mainly because of memory limitations and a considerable increase in processing time. The best solution is to make use of the analytic signal associated with "x(n)" because it has a unilateral spectrum.

The main drawback of the WD is the appearance of artefacts for any signal that is not a single gaussian function (no artefacts appear if the gaussian function is modulating a sinusoidal signal) as shown by equation (2). Such artefacts are composed of relatively high frequency spikes that assume positive and negative values. In order to eliminate them, the original version of the WD must be passed through a two-dimensional lowpass filter. The process is popularly known as "smoothing" and is commonly done by convolving the impure WD with a two-dimensional Gaussian function given by (5), where $\alpha\beta \geq 1$ (Cohen, 1989).

$$L(k,l) = 1/(\alpha\beta).\exp(-k^2/\alpha - l^2/\beta) \qquad (5)$$

As an example, both the impure WD and the smoothed WD of the signal represented by (6) are shown in Figure 1.

$$x(n) = \exp(-a((n-n_1)/N)^2.\cos(2\pi f_1 n/N) + \exp(-a((n-n_2)/N)^2.\cos(2\pi f_2 n/N) \qquad (6)$$

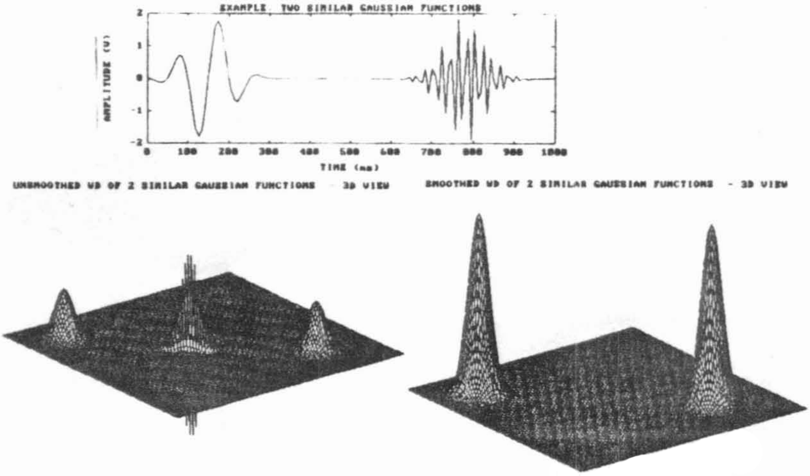

Figure 1.Application of the Wigner Distribution to a test wave form. (All WDs are normalised).

EXPERIMENTAL SETUP

For the implementation of the algorithm, the commercially available MATLAB-386 package was used. The program was run on a 386 IBM-compatible personal computer that has a memory capacity of 7 Mbytes and a clock frequency of 36 MHz.

The EEG signal, band-limited to 70 Hz and pre-recorded on magnetic tape, was digitized at a frequency (fs) of 160 Hz and stored in ASCII-format files. Given x(n) with a total of N samples, the respective WD original matrix has N^2 samples, with a time scale and a frequency scale that go from 0 to (N - 1) and from 0 to fs/2, respectively .

The total processing time depends on two factors: a) the value of N and b) the total number of samples (A.B) that compose the two-dimensional smoothing function L(k,l), where A=kmax- kmin+1 and B=lmax-lmin+1. During the experiments, the total processing time was checked for two distinct cases. First, for N=128 and A=B=11, it took 5 minutes for the final smoothed WD matrix to be obtained. Second, for N=256 and A=B=11, 40 minutes were necessary. Most of the processing time is spent on the smoothing process.

From the results obtained, it was noted that most of the energy of the EEG signal was concentrated below 40 Hz, and a decision was taken to smooth only the lower frequency half of the original WD matrix. In this way, a considerable decrease in the processing time was obtained without the necessity of changing the resolution of the signal in the time domain.

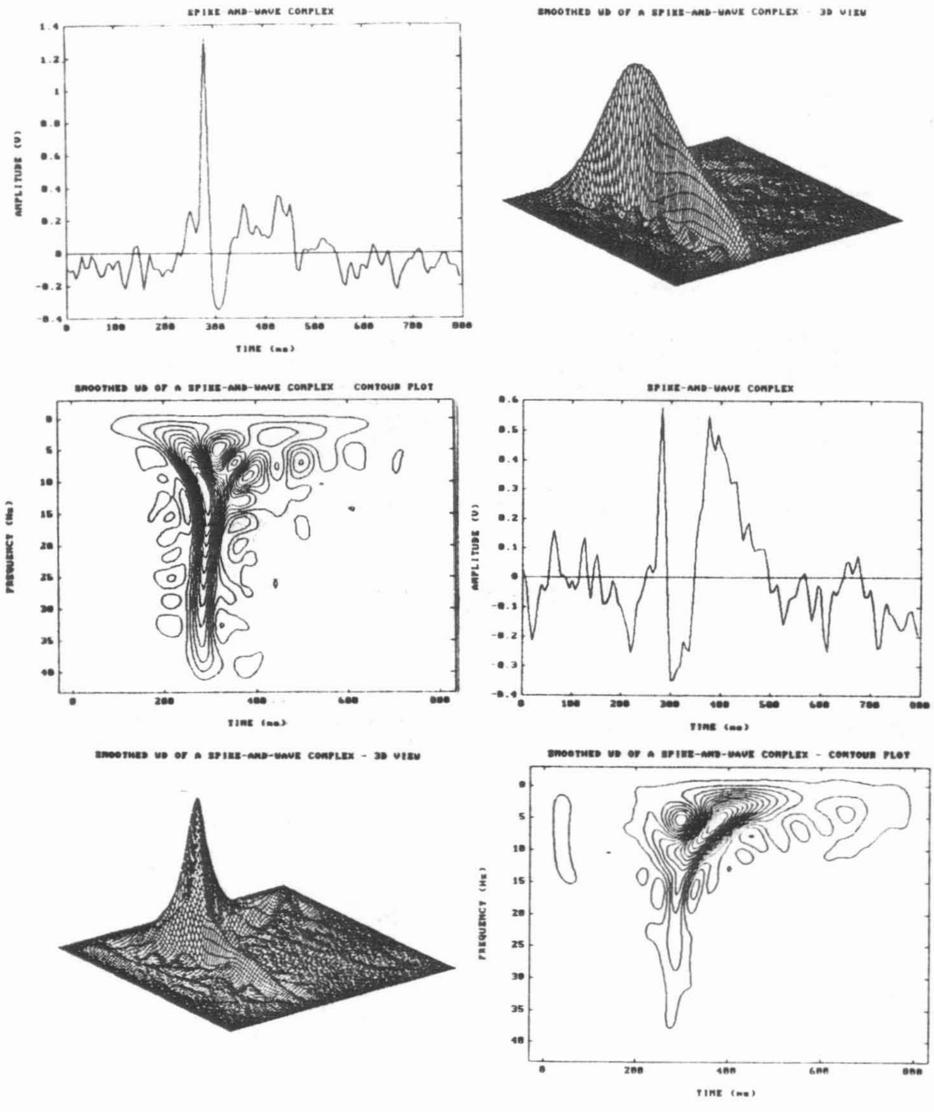

Figure 2. Examples of the application of the Wigner Distribution to spike-and-wave complexes. (All WDs are normalised).

DISCUSSION OF RESULTS

Initial results obtained with the WD are very encouraging. From the two examples given it can be seen that the basic form of the contour plot outputs are very similar for quite different spike-and-wave complexes: both exhibit a chacteristics "T" shape. This has been repeated for all the SAWC inputs we have tried and shows a marked contrast to segments of normal records, for which no particular form is obtained, and for records containing artefacts. The processing time involved on our test setup is quite long and memory capacity can quickly give rise to problems. Various techniques aimed at overcoming these problems are being considered (Boashash & Black, 1987).

CONCLUSIONS

The potential power of the Wigner Distribution in the analysis of EEG records has been demonstrated. From our initial results it would appear that the technique is capable of offering a very reliable means of identifying spike-and-wave complexes from the EEG record, which is our primary area of interest. Further, preliminary studies with records containing artefacts suggest that a high degree of rejection would appear to be possible, although more work is required in this area before any firm conclusions may be drawn.

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REFERENCES

BOASHASH, B. & BLACK, P. J.; An efficient real-time implementation of the Wigner-Ville distribution. IEEE Transactions on Acoustics, Speech, and Signal Processing, 35:(11), 1611-1618, Nov/1987.

CLAASEN, T. A. C. M. & MECKLENBRAUKER, W. F. G.; The Wigner distribution - a tool for time-frequency signal analysis. Part I, II and III. Philips Journal of Research, 35, 217-250, 276-300, 372-389, 1980.

COHEN, L.; On a fundamental property of the Wigner distribution. IEEE Transactions on acoustics, Speech and Signal Processing, 35:(4), 559-561, Apr/1987.

COMLEY, R. A. & BRIGNELL, J. E.; Real-time detection of the epileptic precursor. Journal of Physics E: Scientific Instruments, Vol 14, 1981.

COSTA, E. T. & LEEMAN, S.; The Wigner distribution and its application to non-linear acoustic wave propagation. Revista Brasileira de Engenharia, São Paulo, 6:(2), 220-227, Set/1989.

GOTMAN, J. & GLOOR, P.; Automatic recognition and quantification of interictal epilleptic activity in the human scalp EEG. Electroencephalography and Clinical Neurophisiology, Amsterdam, 41, 513-529, 1976.

NIEDERMEYER, E. & LOPES DA SILVA, F.; Electroencephalography: basic principles, clinical applications and related fields. Munchen, Urban & Schwarzenberg Inc., 1987, 940p.

STELLE, A. L. & COMLEY, R. A.; Portable analyser for real-time detection of the epileptic precursor. Revista Brasileira de Engenharia, São Paulo, 6:(2), 101-107, Set/1989.

VILLE, J.; Theorie et applications de la notion de signal analytique. Cables et Transmission, 2A, 61-74,1948.

WIGNER, E. P.; On the quantum correction for thermo-dynamic equilibrium. Phys. Rew., 40, 749-759, 1932.