QUANTUM NOISE IN CONVENCIONAL RADIOLOGICAL IMAGES

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ABSTRACT -- This paper presents a method for obtaining the signal to noise ratio in conventional radiological images and the signal to noise normalized to the dose absorbed by the patient. These quantities are then optimized both by analytical methods and by computer analysis using realistic X-ray spectra and patient composition and thickness.

INTRODUCTION

Quantum noise is considered the most fundamental cause of the reduction of quality of radiological images Sturm and Morgan (1949), Rimkus and Bailey (1983), Clare et al (1962) and Macoviski (1983) because, unlike other instrumental contributions to the loss of contrast and spatial resolution, quantum noise cannot be eliminated simply by altering instrumental parameters, but only by increasing the fluence of photons incident on the surface of the patient, with an attendant increase in the absorbed dose.

In this paper, we have developed an analytical method that determines the compromise between the desire to keep the risk to the patient small and the demand that photons must be absorbed by the patient to produce an image at the detectors. The method includes the effects of external filters and of the detector efficiency on the image quality and dose absorbed by the patient. The method has been extended to treat digitally Hans and Cullinan (1989), Almeida (1990) developed images such as computerized tomography, and eventually for images obtained from synchrotron radiation sources. We present a method for the determination of the best photon energies to be used in obtaining images for each clinical application not previously reported in the scientific literature.

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METHOD

Supposing a monochromatic source of X-rays, the number of photons that penetrate the patient and are detected may be written by the following expression:

$$C = \Phi t (\Delta r)^2 \epsilon e^{-M} \tag{1}$$

where

Φ_0 = flux of photons incident on the upper surface of the patient

t = exposition time

$(\Delta r)^2$ = area of the detector

ε = efficiency of the detector

e^{-M} = probability that the photons do not interact with the patient

M = absorbance of the patient $M = \int \mu \, dx$

The contrast in the image is given by differences of the absorbances of the patient, and is essentially what the radiologist must have to make his diagnosis. To obtain a difference two counts must be made in adjacent areas and we may define the signal as

$$SIGNAL = \Delta M = \frac{\Delta C}{C} \tag{2}$$

If the two counting rates are not very different, the standard deviation Bevington (1969) of the signal may be given by

$$\sigma_{\Delta M} = \sqrt{\frac{2}{C}} \tag{3}$$

which leads to the signal to noise ratio

$$\frac{S}{R} = \frac{\Delta M}{\sigma_{\Delta M}} = \Delta M \Delta r e^{-M/2} \sqrt{\frac{\Phi_o t \epsilon}{2}} \tag{4}$$

Making the assumption that ΔM and M vary alike with energy, and that ε = 100% is independent of energy, this expression has a maximum at $M = 2$. For $\varepsilon << 100\%$, the maximum will be for $M=3$ because the detector efficiency will vary with energy in the same way as the patient. These values show some correlation with the values of $\mu x = M$ used in a typical radiological service (table I).

TABLE I
Absorbances M used in radiological procedures at HCFMRP-USP

REGION	kVp	THICKNESS (cm)	μ (cm^{-1})	$M = \mu x$
abdomen	80	20	.18	3.6
thorax	90	20	.18	3.6
cranium	80	17	.18	3.1
femur	66	15	.20	3.0
cervical	70	15	.19	2.9
leg	65	12	.20	2.4
knee	55	10	.57	5.7
ankle	55	5	.57	2.9
feet	60	4	.57	2.3
hands	40 a 50	3	.57	1.7

In Equation 4 we note that the signal to noise increases as the square root of the fluence at the surface of the patient.
Not suprisingly, the image quality for a given contrast and spacial resolution improves with the square root of the number of photons incident on the patient. Unfortunately, the dose absorbed by the patient increases linearly with the number of incident photons, and should be taken into account when determining the optimum absorbance in clinical applications.

The dose absorbed Attix (1986) by a surface layer Δx of the patient owing to a flux of photons all with energy E may be written

$$D = \frac{\Phi_o t(\Delta r)^2 \, E\mu_s \, \Delta x}{(\Delta x \Delta r^2)\rho} = \Phi_o t E(\mu/\rho)_s \quad J \ kg^{-1} \tag{5}$$

Where is the mass attenuation coeficient at the surface of the patient.

For clinical applications instead of Equation 4, we have the following expression to be maximized:

$$\frac{S/R}{D} = \frac{\Delta r \Delta M \sqrt{\epsilon/2}}{\sqrt{\Phi_o t} \, E(\mu/\rho)_s} e^{-M/2} \tag{6}$$

For the same extremes in detector efficiency, this expression has a maximum value at M=2/3 for a saturated detector, and M=5/3 for a thin detector.

We note that Equation 6 shows clearly that increasing the fluence decreases the signal to noise ratio once it is normalized to the dose absorbed by the patient. This puts the burden on the radiologist, not only to choose the optimum absorbance by choosing the appropriate kVp, but also to use only the minimum fluence necessary for his diagnosis.

Equations 4 and 6 are developed for monochromatic X-ray sources. In reality one has a continuous X-ray spectra modified by internal and external filters between the generator and the patient. In place of the Equations 4 and 6 we have the following expressions where each energy dependent expression is replaced by its average over the X-ray spectrum at the surface of the patient:

$$S/R = \frac{\Delta C/C}{\sigma_{\Delta C/C}} = \Delta r \sqrt{\frac{t}{2}} \, \frac{\int (d\Phi/dE) dE \epsilon \Delta M e^{-M} \quad \sqrt{\int^{T_o} (d\Phi/dE) dE}}{\sqrt{\int (d\Phi/dE) dE \epsilon e^{-M}}} \tag{7}$$

$$\frac{S/R}{D} \propto \frac{\int^{T_o} (d\Phi/dE) dE \epsilon M e^{-M} \sqrt{\int^{T_o} (d\Phi/dE) dE}}{\sqrt{\int^{T_o} (d\Phi/dE) dE \epsilon e^{-M}} \, \int^{T_o} (d\Phi/dE) dE E(\mu/\rho)_s} \tag{8}$$

NUMERICAL OPTIMIZATION OF THE SIGNAL TO NOISE RATIOS

Expressions like 9 and 10 may be maximized relative to the absorbance. The maximum can easily be achieved by numerica method for each clinical application since one has available for computation the counting rates in the detectors, and the flux incident at the surface of the patient.

The simulation of the X-ray spectra has been made through the use of the theory of Birch and Marshall (1978) furnished with data on the generator high voltage, the anode angle, the intrinsic filtration, and the material and thickness of additional external filters.

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