# AN INVESTIGATION OF SIGNAL PHASE IN A VISUAL EVOKED POTENTIAL

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David M. Simpson

**ABSTRACT** – It has repeatedly been claimed by various authors that signal phase is more closely related to signal shape than is the amplitude spectrum. This is confirmed in an experimental study for the case of a visual evoked potential. It is shown that this may generally be expected in signals with a smooth amplitude spectrum. It is concluded that in such signals, quantitative analysis should therefore not be restricted to the amplitude spectrum. Some of the problems involved in phase analysis are also discussed and the application of tapered data windows in phase estimation is experimentally studied.

## INTRODUCTION

The discrete Fourier transform is one of the most commonly used techniques in the analysis of signals. This converts a signal of N sample length into a complex spectrum of an equal number of harmonics. These complex numbers can be converted into polar form, giving an amplitude and a phase spectrum. To date, by far the largest part of quantitative signal analysis based on spectral parameters has concentrated on the amplitude spectrum - this applies to both biomedical and non-biomedical signals. However, it has been repeatedly observed that signal phase holds more information with respect to signal shape than does the amplitude. For example, Sayers et al. (1974a, 1974b) claimed this for auditory evoked potentials, Infantosi and Sayers (1989) analysed epidemiological data considering phase, Oppenheim and Lim (1981) obtained good results in reconstructing speech signals and images using only phase information and Steinberg (1987) showed for the example of microwave images, that greater precision is required in phase than in amplitude data.

It therefore appears that if in such signals spectral analysis is restricted to the amplitude, a significant part of the information is discarded. In quantitative studies, phase might lead to signal parameters that are more strongly related to signal shape and therefore have greater diagnostic value. However, little has been published on phase-spectral analysis (and estimation) and out theoretical knowledge is very limited. Primarily it must be established under

Prof. Visitante do Programa de Engenharia Biomedica- COPPE/UFRJ, CP. 68510, CEP 21945, Rio de Janeiro - RJ.

what circumstances phase is vital and techniques for its estimation studied.

It is therefore the aim of this work study to investigate the relative importance of signal phase and amplitude in signals of the shape of the visual evoked potential (VEP). In addition this work serves to illustrate some of the phenomena and problems involved in phase analysis and aims at improving the theoretical understanding. In the estimation of signal phase, tapered windows have been used by various investigators, and their effect on the phase-estimates for the VEP under study is demonstrated.

**Signal Characteristics of the VEP**

For the present investigation, an idealized form of a pattern onset VEP, as given by Barber(1984), was employed. Thus the analysis can concentrate on the essential elements in the shape of the VEP and the results are not distorted by noise or other forms of contamination. The signal covers a period of approximately 500ms after stimulus onset and was digitized to 512 samples. Fig. 1 shows this signal (VEP1). Note that the spikey shape resulted from the subsampling and linear interpolation of the plotting routine used in this figure and that the signal processed was smooth.

A second signal (VEP2) was generated from VEP1 by removing the flat periods before response onset and after its termination. This period was then resampled to give again 512 samples, as shown in Fig. 1.

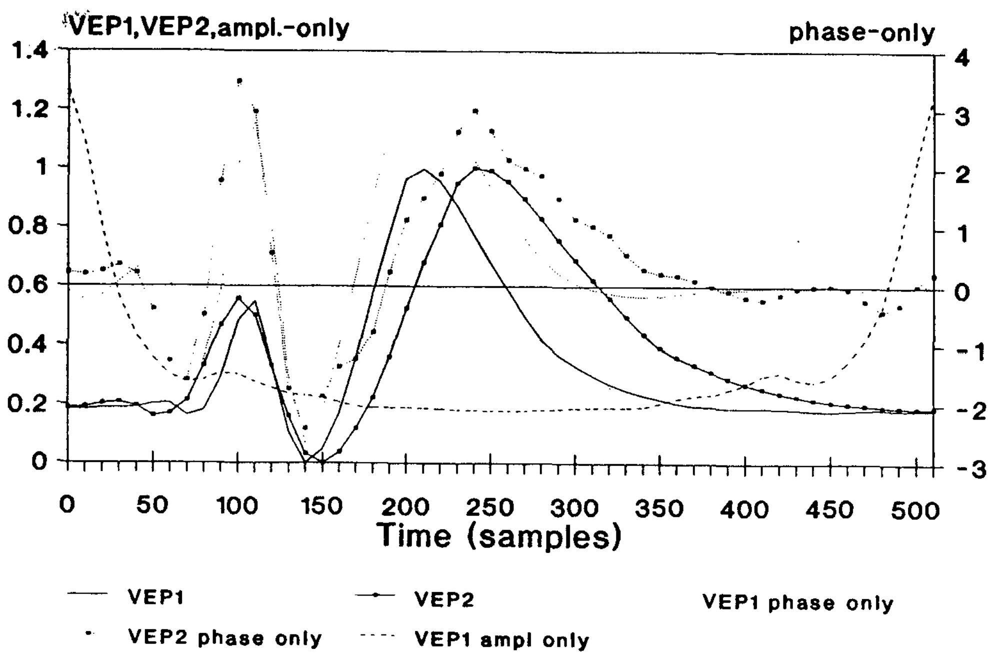


Fig. 1. The signals VEP1 and VEP2 and amplitude- and phase-only reconstructions.

The amplitude and phase spectra of these signals were calculated using the discrete

Fourier transform (DFT) and are given in Fig.2. As VEP2 is approximately a time-scaled version of VEP1, the spectra are approximately frequency scaled versions of each other, as follows from the scaling theorem (Papoulis, 1984). The amplitude spectra show that, as expected, only the first few harmonics are of significant magnitude and clearly it lis also the phase of only these that is of interest in the analysis below.

The phase spectra illustrate one of the main difficulties in phase analysis, known as 'phase wrapping'. The phase-angles were calculated only in the range of $\pm \pi$, leading to discontinuities, seen for example at the 2nd and 7th harmonic for VEP2. It is clear that through 'phase unwrapping' a smoother phase spectrum could be achieved (up to the 14th harmonic for VEP2 and 18th for VEP1).

In order to reduce the incidence of phase wrapping and to be able to compare the phase of VEP1 and VEP2, the signals were delayed and aligned before calculating the spectra. As is well known, delay introduces linear phase shift

$$y(i) = x(i-d) <==> Y(i) = X(i) \exp(-j (2\pi/N)\, i\, d),$$

where x(i), y(i) and X(i) and Y(i) are the discrete signals and their DFTs respectively, and N is the length of the data. By delaying the signals, the phase difference between successive harmonics can be reduced (or increased). For the plots above, an 'optimal' delay was calculated so as to minimize the phase- difference between successive harmonics (up to the 15th). This corresponds to the 'most symmetrical position' defined by Infantosi and Sayers (1989), but was calculated using the whitened complex spectra rather than the phase-values.

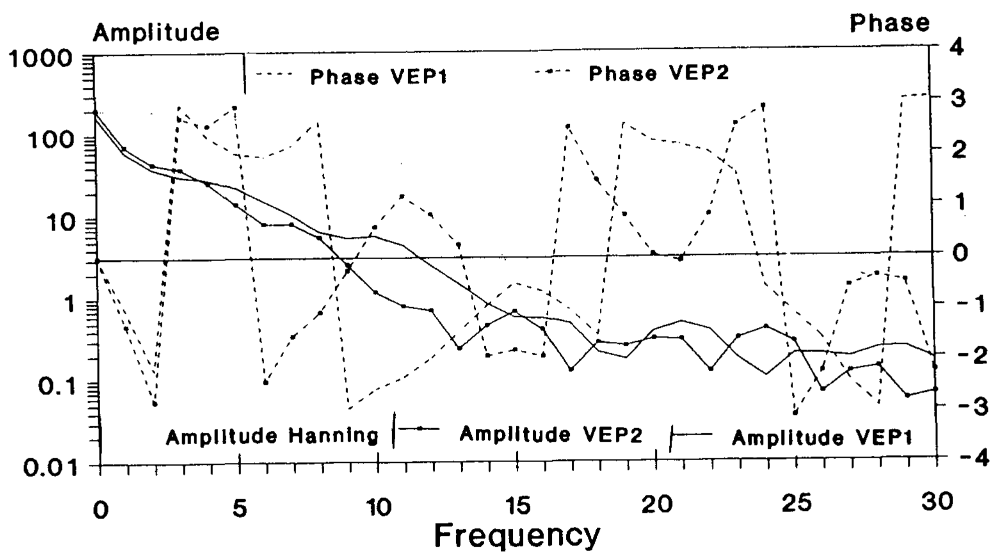


Fig. 2. Amplitude and phase spectra. Phase in radians.

**Amplitud and Phase-Only Reconstruction**

In order to evaluate the relative importance of phase and amplitude, it may be attempted to reconstruct the signals using only the amplitude and only the phase information respectively. The result of amplitude-only reconstruction, i.e. setting all phase values to zero and using the amplitude spectrum of VEP1, is shown in Fig. 1. The signal is very distorted.
As has already been observed, the amplitude is negligible at high frequencies and therefore the phase of only the lower harmonics should be employed in phase-only reconstruction. In Fig. 3.a successively higher harmonics were included in the inverse DFT, each with unit amplitude but the phase of VEP1. As the number of harmonics is increased (moving to the top of the plot), the basic shape of VEP1 emerges, but then becomes almost drowned by unwanted oscillations. This may have been expected a result of the rectangular shape of the amplitude spectrum chosen, which is known to lead to Gibbs oscillations (Oppenheim and Schafer, 1975). It is also well known that this phenomenon can be reduced by providing a smoother change in the amplitude spectrum. To this end, an amplitude spectrum in the shape of a raised cosine (Hanning window) was then employed, leading to the much improved results shown in Fig. 3.b. The shape of VEP1 clearly emerges.

In Fig. 1, one of these reconstructions (including up to the 15th harmonic) is shown together with the original VEP1. Here it may be confirmed that the basic shape and the location of peaks and troughs are approximately maintained. Using exactly the same amplitude spectrum but the phase of VEP2, the shape of this signal emerges. The phase therefore appears to contain most information regarding latencies and what has been lost in this phase-only reconstruction is mainly the relative amplitude of peaks and troughs.

In Fig. 2, the amplitude spectrum of VEP1 and VEP2 is shown, together with the Hanning spectrum employed here. It is noted that those for VEP1 and VEP2 are more similar to each other than they are to that of the phase-only signals. With this latter amplitude spectrum however good reconstruction was achieved for both VEP1 and VEP2. This emphasizes that it is primarily the phase that determines the difference between the signals of different latencies rather than their amplitude.

This conclusion suggests that in quantitative analysis of VEP data, attention should not focus exclusively on the amplitude spectrum, but phase parameters should also be considered. The question then arises, to what extent this result may be generalized to other signal shapes and furthermore, what signal characteristics determine the relative importance of amplitude and phase.

Oppenheim and Lim (1981) observed that reconstructions appear to work best with a smooth spectrum. This can be justified from theoretical considerations:
Let X(w) be the original (continuous) spectrum and Y(w) the phase-only version. Then

$$Y(w) = |Y(w)|.\exp(j.\arg\{X(w)\}),$$

where arg{.} is the phase angle. This may be considered a linear filtering operation,

$Y(w) = H(w).X(w)$, where

$H(w) = |Y(w)|/|X(w)|$.

The filter has therefore zero phase and $H(w) \geq 0$, by definition. The impulse response of the filter h(t) is symmetrical and has the largest positive peak at t=0. If the impulse response h(t) is strongly concentrated near zero, then the effect of this filtering operation will be very localized and is not expected to greatly move the position of large peaks and troughs. Phase-only reconstruction is therefore expected to be effective, and the phase may be considered the more significant aspect of the spectrum.

A measure of the 'spread' of h(t) may be given by its second moment:

$$M = \int_{-\infty}^{\infty} (h(t).t)^2 dt.$$

By applying the derivative law and Parseval theorem (Papoulis, 11984) it can easily be shown that this is equivalent to

$$M = (1/2\pi)\int_{-\infty}^{\infty} |H'(w)|^2 dw,$$

where H'(w) is the first derivative of H(w).

A smooth function H(t) therefore will lead to a 'short' or 'compressed' impulse response h(t) - as also observed by Oppenheim and Lim (1981). The function $|Y(w)|$ is chosen as a smooth one and it follows that the 'spikeyness' of H(w) is determined primarily by the spikeyness of $|X(w)|$. Consequently a smooth $|X(w)|$ leads to a smooth $|H(w)|$, a 'short' h(t) and good phase-only reconstruction is expected.

## TAPERED DATA WINDOWS AND PHASE ESTIMATION

In the estimation of power spectra, the use of tapered data windows is well established (Harris, 1978). Such windows have also been applied in the estimation of signal phase (Sayers et al. 1974 a and b). It appears however that their use has not been rigorously justified, nor optimal windows for this application developed. It is shown here that an inappropriate choice of window-function can lead to very poor results.

In order to evaluate the effectiveness of tapered data windows, a series of Tukey (cosine-taper) windows were applied to VEP2, the 512 point DFT performed and the phase-estimates thus obtained compared to ones calculated from the full length VEP2. In order to avoid problems of phase-wrapping, an error measure was employed that is based on the complex plane. The spectra of both the original and windowed signals were whitened (unit amplitude for all harmonics) and the mean-square difference calculated over the first 10 harmonics. These mean-square errors, normalized by the power of the whitened original data, are shown in Fig. 4, for a range of window-lengths (end-points of the window) and Tukey-window rise-times (r, expressed as a fraction of window-length). As expected the rectangular window performed best as the window-length approached the true length of the signal. With a reduction in window length, the rectangular window introduces a step in the data and tapering improves the results. It is interesting to note however, that mid-range taper ($r=0.3$) performed best in this case and only for the shortest window lengths can Hanning windows be recommended. Over most of the range, Hanning windows gave very poor results. While the Hanning window eliminated the abrupt step in the data, it also greatly distorted the shape of the remaining segment, and since phase is sensitive to shape, large phase errors resulted.

It is clear that the choice of a best window must depend on the specific signal characteristics and a 'universally' optimal window cannot be given.

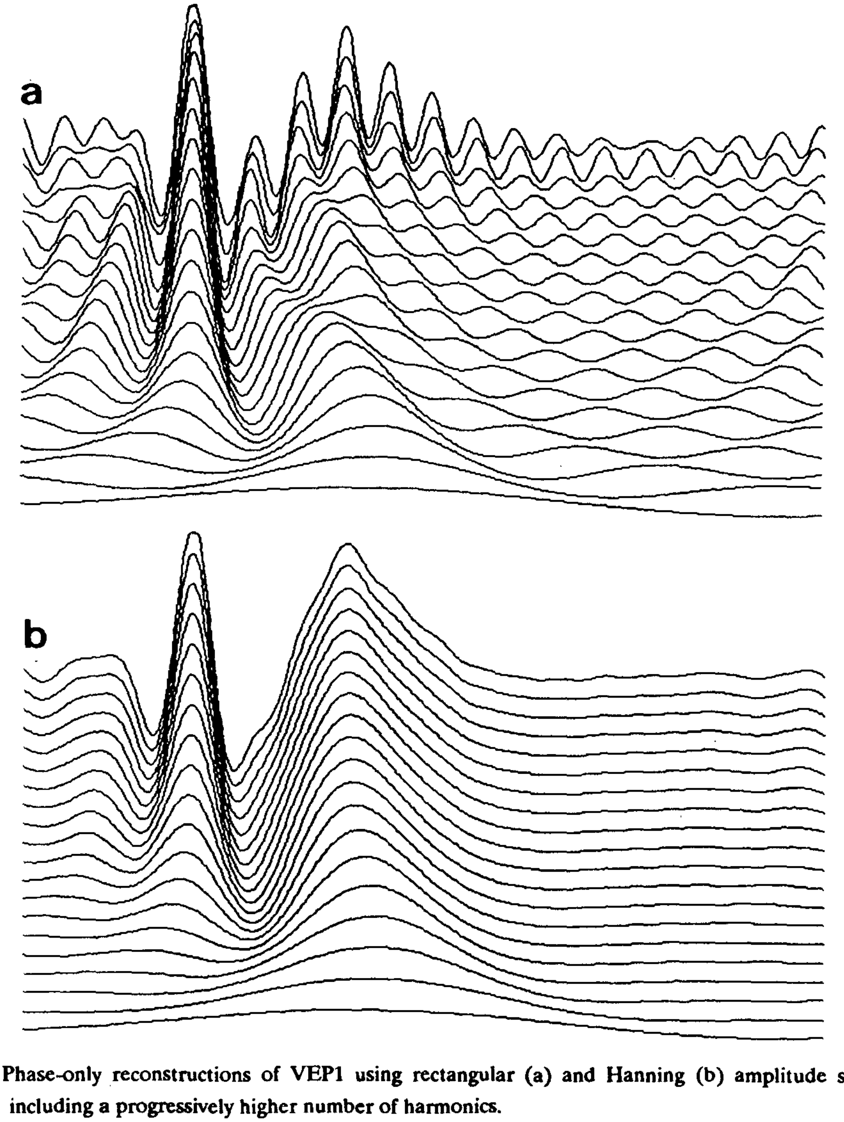


Fig. 3. Phase-only reconstructions of VEP1 using rectangular (a) and Hanning (b) amplitude spectra, including a progressively higher number of harmonics.

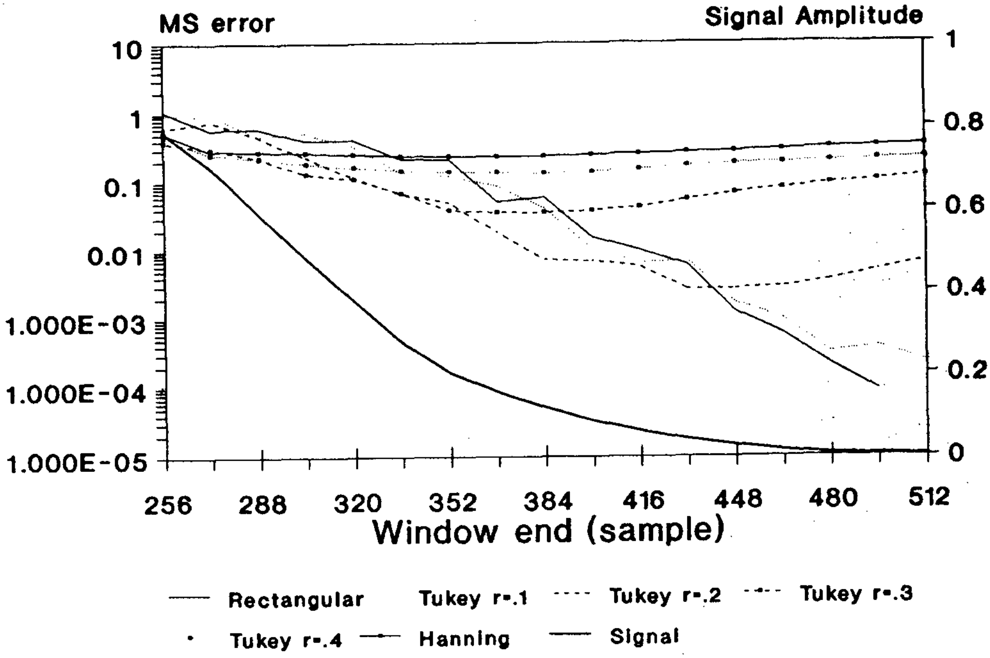


Fig. 4.Mean-square phase errors as a result of applying data windows starting at sample 0. The signal at the window end-point is also shown. Tukey data windows with a rise- time of r=0 (rectangular) to r=0.5 (Hanning).

## CONCLUSIONS

The above work has shown that in VEP-like signals, phase is closely linked to signal shape, in particular the location of peaks and troughs. Employing the same amplitude spectrum and only modifying the phase, the shape of two different signals was accurately obtained. It may be generalized that a smooth amplitude spectrum indicates that such phase-only reconstruction will be effective. Finally, in the estimation of signal phase, tapered data windows can improve the results when rectangular windows would lead to a large step. However, Hanning windows were found to perform poorly.

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